

City College

Tutorial Assessment-2021

B.Sc. / MTMG

Sem-1 / Paper-GE1

Full marks-15

Answer any three questions and submit answer scripts as directed. $5 \times 3 = 15$

1) In the complex plane, if z_1, z_2, z_3 be the vertices of an equilateral triangle ABC, then verify that $\frac{1}{z_1 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0$.

2) For any two complex numbers z_1 and z_2 , prove that

$$2|z_1 + z_2| \geq (|z_1| + |z_2|) \left| \frac{z_1}{|z_1|} + \frac{z_2}{|z_2|} \right|$$

3) If $z = \frac{(a-b) + (a+b)i}{(a+b) + (a-b)i}$, where a, b are two positive real numbers then prove that

$$\log z = i \tan^{-1} \frac{2ab}{a^2 - b^2}, \text{ if } a < b$$

$$= i \left(\pi + \tan^{-1} \frac{2ab}{a^2 - b^2} \right), \text{ if } a > b$$

$$= i \frac{\pi}{2}, \text{ if } a = b.$$

4) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 2ay$, a being a parameter.

5) (a) If the two straight lines represented by $ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$ be at right angles, then show that $a = 2$ & $c = -3$.

(b) Show that, the locus of the point of intersection of a pair of perpendicular tangents to an ellipse is a circle.