

Answer any three questions: $3 \times 5 = 15$

1) Solve the differential equation:

$$(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x) \frac{dy}{dx} + 16y = 8(1+2x)^2 \quad (5)$$

2) (a) Test the convergence of the series

$$1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \dots \rightarrow \infty$$

(b) For all $x > 0$, prove that

$$\frac{x}{1+x} < \log(1+x) < x \quad (3+2)$$

3) Solve the system of congruences using Chinese Remainder Theorem:

$$x \equiv 1 \pmod{3}, \quad x \equiv 2 \pmod{4}, \quad x \equiv 3 \pmod{5} \quad (5)$$

4) Prove that, $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

Interpret this result geometrically. $(2+3)$

5) Find the correct check digit for the ~~UPC~~ incomplete UPC 890102300003. (5)

6) Express the Boolean expression

$$(xy' + xz)' + x' \text{ to its DNF and CNF.} \quad (5)$$