City College

Internal Assessment 2021

Physics (Hons.) CBCS Semester 4

Paper: CC-8: Mathematical Physics III

Time: 1 Hour; Full Marks: 20

Answer any 10 questions. Each question carries 2 marks.

1. Find the singularities of the function $f(z) = \frac{1}{1-z} - \frac{1}{1+z}$.
2. For $f(z) = u + iv$, given that $u = x^2 - y^2$, determine $f(z)$ up to an additive constant using Cauchy-Riemann conditions.
3. Find the residue of $f(z) = \frac{\tan z}{z^2}$ at $z = 0$.
4. Show that if $f(z)$ is an analytic function, then $\frac{df}{dz^*} = 0$.
5. Does the density of an object change as its speed increases? If yes, by what factor?
6. Consider the line element

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

   Write down all the non-vanishing elements of the metric $g_{\mu\nu}$.
7. What do you mean by space-like and time-like vectors?
8. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum P. If c denotes the velocity of light in vacuum, what is the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v?
9. Consider a Lorentz transformation in a 2-dimensional space-time given by

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(-\beta ct + x), \end{aligned}$$

   where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ and $\beta = \frac{v}{c}$. Write down the matrix $\Lambda(\beta)$ of the above transformation and check if $\Lambda(\beta)$ is orthogonal.
10. Show that 4-velocity and 4-acceleration are orthogonal to each other: $u^\mu a_\mu = 0$.
11. The Lagrangian of a particle of mass m and charge q in an electromagnetic field is given by

$$L = \frac{1}{2}mv^2 - q\left(\Phi - \vec{v}\cdot\vec{A}\right),$$

where Φ and $\vec{A}$ are the scalar and vector potentials respectively. Now consider the transformations:

$$\Phi \to \Phi - \frac{d\chi}{dt}, \qquad \vec{A} \to \vec{A} + \vec{\nabla}\chi.$$

Does the Lagrangian remain unchanged under this transformation? Does the equation of motion remain unchanged? Explain your answer.

12. Show that the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$ represents a free particle.

13. A particle moves in a potential $V = x^2 + \frac{1}{2}y^2 + z^2$. Which component(s) of the angular momentum is/are conserved?

14. Consider two non-interacting systems A and B with Lagrangians L_A and L_B. Now consider the composite system $A \cup B$ with Lagrangian $L_{A\cup B}$. Is $L_{A\cup B} = L_A L_B$ a valid Lagrangian of the system? Justify your answer.

15. A system is described by the Lagrangian $L = \lambda \dot{q}_1 \dot{q}_2$. If p_1 and p_2 be the momenta canonical to q_1 and q_2 respectively, find the Hamiltonian of the particle.

Answer scripts must be emailed to ***sem4hcityphysics@gmail.com*** *within 15 minutes of the end of the examination.*