

CITY COLLEGE
Internal Examination 2020-2021
Physics (Hons.) CBCS Semester 1
Paper: PHSA CC1
Topic: Mathematical Physics 1
Time: 1 Hour; Full Marks: 20

Answer any ten questions. Each question carries two marks.

1. What do you mean by linear independence of vectors?
2. Show that the product of two orthogonal matrices is orthogonal.
3. What is the value of the series

$$\left(1 - \frac{1}{2!} + \frac{1}{4!} - \dots\right)^2 + \left(1 - \frac{1}{3!} + \frac{1}{5!} - \dots\right)^2?$$

4. Show that $\sum_{n=1}^{\infty} n!x^n$ has zero interval of convergence.
5. Show that any square matrix can be written as a sum of symmetric and antisymmetric matrices.
6. If $\hat{\rho}$ be the unit vector along the radial direction in plane polar coordinates $\{\rho, \phi\}$, evaluate $\oint \hat{\rho} d\phi$ along a unit circle centered on the origin.
7. A matrix P satisfies $P^2 = P$. Show that its eigenvalues are 0 and +1.
8. Explain whether the series $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ converges or diverges.
9. If A is a (2×2) matrix, show that its eigenvalues λ satisfy $\lambda^2 - \lambda \text{Tr}[A] + \det[A] = 0$.
10. Consider the linear differential equation $y'(x) = xy$. If $y = 2$ at $x = 0$, then what is the value of y at $x = 2$?
11. Consider the function $x|x|$. Sketch this function in the range $(-5, +5)$. State whether the function is odd or even.
12. Evaluate $\int \mathbf{V} \cdot d\mathbf{s}$, where $\mathbf{V} = x \cos^2 y \hat{\mathbf{x}} + xz \hat{\mathbf{y}} + z \sin^2 y \hat{\mathbf{z}}$ over the surface of a sphere with center at the origin and radius three units.

Answer scripts must be emailed to sem1hcityphysics@gmail.com within 15 minutes of the end of the examination.