

2020

## PHYSICS — HONOURS

Paper : CC-5

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any four** from the rest.1. Answer **any five** questions :

2×5

(a) Show that any function  $f(x + t)$  is a solution of the equation

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t},$$

when  $u = u(x, t)$ .(b) Using Gamma function integral, prove that  $0! = 1$ .(c) Using generating function for Legendre Polynomial, prove that  $P_{2l+1}(0) = 0$ .(d) Find the indicial equation for the differential equation  $(1-x^2)\frac{d^2y}{dx^2} + px\frac{dy}{dx} + qy = 0$ , where  $p$  and  $q$  are constants.

(e) Find the non zero Fourier coefficients of

$$f(x) = \cos^2x, \quad 0 \leq x < 2\pi.$$

(f) Find the Fourier transform of the Dirac  $\delta$ -function,  $\delta(x - a)$ .**Or,**For a generalised coordinate  $q$ , the Lagrangian is given by

$$L = \alpha\dot{q}^2 + \beta q\dot{q} - \gamma \sin q,$$

where  $\alpha, \beta, \gamma$  are constants. Find the Hamiltonian  $H$ .(g) For a random variable  $X$ , one finds  $\langle X \rangle = 2$  and  $\langle X^2 \rangle = 8$ . Find the standard deviation  $\sigma_x$ .**Or,**

For the Hamiltonian

$$H = ap^2 + bx^2 - cxp$$

(where  $a, b, c$  are constants), find the Hamilton's equation of motion.**Please Turn Over**

2. (a) Find the probability density function  $f(x)$  for the position of a particle executing SHM on  $(-a, a)$  along the  $x$ -axis.
- (b) Let  $X$  be a random variable having a normalized density function

$$f(x) = \begin{cases} Ce^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find :

- (i) the value of  $|C|$  where  $C$  is a constant.
- (ii) the variance  $\sigma^2$ .
- (iii) the probability  $P(1 \leq x \leq 2)$ . 3+(2+3+2)

**Or,**

- (a) Write Lagrange's equation in cylindrical co-ordinates for a particle of mass  $m$ , moving in the gravitational potential  $V = mgz$  starting from the Lagrangian.
- (b) Is there any cyclic co-ordinate? If yes, find the corresponding conserved quantity.
- (c) Given two points  $P_1$  and  $P_2$  (not too far apart), we draw a curve joining them and revolve it about the  $x$ -axis. Find the curve for which the surface area is minimum. 3+2+5
3. (a) Consider a cycloid with parametric equation  $x = a(\theta + \sin\theta)$ ,  $y = a(1 - \cos\theta)$ . Show that the time for a particle to slide without friction along the curve from  $(x_1, y_1)$  to origin is independent of starting point. (use beta function to evaluate the integral).
- (b) Prove  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ , symbols are usual. 5+5

4. (a) Given  $f(x) = \begin{cases} 0, & 0 < x < l \\ 1, & l < x < 2l \end{cases}$  .

Expand the function in Fourier series of period  $2l$ .

- (b) Let a function  $f(x)$  be expanded in Fourier series. Show that average of  $[f(x)]^2$  over a

$$\text{period} = \left(\frac{a_0}{2}\right)^2 + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 + b_n^2 .$$

- (c) Using the result of (b), show that using  $f(x) = x$ ,  $-1 < x < 1$  the infinite sum  $\sum_1^{\infty} \frac{1}{n^2} = \pi^2/6$  .

4+3+3

5. (a) Use Rodrigue's formula

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l.$$

Show that  $P_l(1) = 1$ .

- (b) Using (a), find  $P_3(x)$ . Plot  $P_3(x)$  as a function of  $x$  where  $-1 \leq x \leq 1$ .

- (c) Calculate  $\int_{-1}^{+1} x^2 P_3(x) dx$ . 3+(2+2)+3

6. Consider the differential equation  $\frac{d^2 y(x)}{dx^2} + y(x) = 0$

- (a) Check whether  $x = 0$  is an ordinary point or a regular singular point.  
 (b) Find the indicial equation.  
 (c) From the indicial equation, find two linearly independent solutions of the given differential equation. 2+2+(3+3)

7. (a) Consider the one-dimensional wave equation for waves propagating along a string of length  $l$ . Its ends are fixed at  $x = 0$  and  $x = l$ . The string is struck by a fine hammer such that it has an initial displacement zero everywhere but has an initial velocity  $v$  at  $x = \frac{3l}{4}$ .

Find the solution of the wave equation in this case.

- (b) Consider

$$\nabla^2 \phi(r, \theta, \phi) = f(r).$$

Using separation of variables, write down three ordinary differential equations. Solve the equation for  $\phi$  coordinate. 6+3+1

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