2020

MATHEMATICS — HONOURS

Paper : DSE-A-1

(Industrial Mathematics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct answer with proper justification / explanation for each of the multiple choice question given below : (For each question, one mark for each correct answer and one mark for justification) : 2×10

(a) In the CT scan, we use... beams to detect the suspected broken bone locations within the medium.

(i) Hard X-ray (ii) Soft X-ray

(iii) Electron (iv) γ-ray.

(b) Differential equation known as Beer's law is an

(i) ordinary 2nd order linear differential equation

(ii) ordinary 2nd order nonlinear differential equation

(iii) ordinary 1st order linear differential equation

(iv) ordinary 1st order nonlinear differential equation.

(c) The definition of a periodic function, is given by a function which

(i) has a period $T = 2\pi$ (ii) satisfied $f(t + T) = f(t)$

(iii) satisfied $f(t + T) + f(t) = 0$ (iv) has a period $T = \pi$.

(d) A signal $x(t)$ has a Fourier Transform $X(\omega)$. If $x(t)$ is real and odd Function of t, then $X(\omega)$ is

(i) a real and even function of ω

(ii) an imaginary and odd function of ω

(iii) an imaginary and even function of ω

(iv) a real and odd function of ω.

(e) A line $\mathcal{L}_{t,\theta} = \{(t \cos\theta - s \sin\theta, t \sin\theta + s \cos\theta) : -\infty < s < \infty\}$ is perpendicular to the unit vector $\mathbf{n}$. Then

(i) $\mathbf{n} = (\cos\theta, \sin\theta)$ (ii) $\mathbf{n} = (-\cos\theta, \sin\theta)$

(iii) $\mathbf{n} = (\cos\theta, -\sin\theta)$ (iv) $\mathbf{n} = (-\cos\theta, -\sin\theta)$.

(f) The value of the integral $\int_{-\infty}^{\infty} e^{-Ax^2} dx$ is

(i) $\frac{\pi}{A}$ (ii) $\sqrt{\frac{\pi}{A}}$

(iii) $\frac{1}{A}$ (iv) $\frac{1}{\sqrt{A}}$.

(g) If $\delta(x)$ be a delta function, such that $\int_{-\infty}^{\infty} \delta(x)dx = 1$, then the Fourier transform of $\delta(x)$ is

(i) 1 (ii) $\frac{1}{\delta(1)}$ (iii) $\delta(1)$ (iv) $\sqrt{\delta(1)}$.

(h) If the 2×2 matrix X satisfies the equation $X\begin{pmatrix} 4 & 7 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$, then $X =$

(i) $\begin{pmatrix} -6 & 4 \\ 13 & -10 \end{pmatrix}$ (ii) $\begin{pmatrix} -6 & 5 \\ 13 & -10 \end{pmatrix}$ (iii) $\begin{pmatrix} -6 & 4 \\ 12 & -10 \end{pmatrix}$ (iv) $\begin{pmatrix} -6 & 4 \\ 13 & -1 \end{pmatrix}$.

(i) If $\mathcal{R}f(t, \theta)$ denotes the Radon transform of f, which one of the following is true?

(i) $\mathcal{R}(\alpha f + \beta g) = \alpha^2 \mathcal{R}f + \beta^2 \mathcal{R}g$ (ii) $\mathcal{R}(\alpha f + \beta g) = \alpha \mathcal{R}f + \beta \mathcal{R}g$

(iii) $\mathcal{R}(\alpha f + \beta g) = (\alpha-1)\mathcal{R}f + (\beta-1)\mathcal{R}g$ (iv) $\mathcal{R}(\alpha f + \beta g) = \mathcal{R}f + \mathcal{R}g$.

(j) If f is continuous on the real line, $\int_{-\infty}^{\infty} |f(x)|dx < \infty$ and $\mathcal{F}$ denotes the Fourier transform of f, then

(i) $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f^{-1}(x)\ \forall x$ (ii) $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f^2(x)\ \forall x$

(iii) $\mathcal{F}^{-1}(\mathcal{F}f)(x) = 2f(x)\ \forall x$ (iv) $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f(x)\ \forall x$.

Unit - I

2. Answer ***any two*** questions :

(a) In CT scan which kind of X-ray is used and why? Explain with suitable example. 5

(b) (i) Let $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + 1$. Find $(f^{-1})'(28)$.

(ii) Find all complex numbers z such that $|z| = 1$ and $|z^2 + \bar{z}^2| = 1$. 2+3

(c) If A be a real matrix, then prove that all the eigenvalues $A^T A$ are non-negative real numbers and the corresponding eigenvectors are orthogonal. 5

(d) Solve the differential equation $x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$. 5

Unit - II

3. Answer ***any two*** questions : 5×2

(a) What do you mean by an inverse problem of a mathematical problem? Explain it with an example.

(b) Write down the inverse problem of the direct problem : Compute the eigenvalues of the given matrix $A + D$, where A being a real symmetric matrix of order $n \times n$ and D is a $n \times n$ diagonal matrix.

(c) Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$.

(d) Solve the differential equation, $\frac{dy}{dx} + \frac{x}{1-x^2} y = x\sqrt{y}$.

Unit - III

4. Answer ***any one*** question : 5×1

(a) State Beer's law on X-ray beam. Write its differential equation form. Establish the result

$$\int_{x_0}^{x_1} A(x)dx = ln\left(\frac{I_0}{I_1}\right)$$

where $A(x)$ is the attenuation coefficient function and $I(x)$ is the intensity of the X-ray beam.

(b) An X-ray beam $A(x)$, propagates in a medium is defined by

$$A(x) = \begin{cases} 1 - |x|, & \text{if } |x| \le 1, \\ 0, & \text{if } |x| > 1 \end{cases}.$$

Find the intensity $I(x)$ of this beam, with the initial condition $I(-1) = 1$.

Unit - IV

5. Answer ***any one*** question : 5×1

(a) Find the Random transform of the function

$$f(x, y) = \begin{cases} 1 - \sqrt{x^2 + y^2}, & \text{if } x^2 + y^2 \le 1 \\ 0 & \text{if } x^2 + y^2 > 1 \end{cases} \text{ on a line } \mathcal{L}_{t,\theta}.$$

(b) Write a short note on Shepp–Logan Mathematical phantom.

Unit - V

6. Answer ***any one*** question :

(a) Define back projection. Prove that the back projection is a linear transformation. 2+3

(b) Give an example of back projection in the context of medical imaging. 5

Unit - VI

7. Answer ***any two*** questions : 5×2

(a) Write a short note on CT scan within 500 words.

(b) Describe an algorithm of CT scan machine.

(c) Find the Fourier transformation of the function $(ax^2 + bx + c)e^{-dx^2}$, $-\infty < x < \infty$, where $a, b, c, d > 0$.

(d) If f be a continuous functions, such that $\int_{-\infty}^{\infty} |f(x)|\, dx < \infty$, then prove that $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f(x)$ for all x, where $\mathcal{F}f$ and $\mathcal{F}^{-1}f$ denote respectively the Fourier and inverse Fourier transform of f.
