2020

PHYSICS — HONOURS

Paper : DSE-B-1

(Astronomy and Astrophysics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1 & 2** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Explain what astronomers mean by the term 'Standard Candle'. Give examples used in cosmology.

(b) The parallaxes of stars A and B are measured as 0.1 arcsec and 0.01 arcsec respectively. Compare their distances.

(c) Why do Fusion reactions in the stellar core stop at iron?

(d) Write down the virial theorem for a gravitationally bound system. Explain each term.

(e) What is cosmological redshift? How is it different from Doppler shift?

(f) What is meant by opacity? Mention the factors contributing to it.

(g) What is the physical significance of Jeans Mass?

2. Answer ***any three*** questions :

(a) What do you mean by the light gathering power and resolving power of a telescope? Compare the resolving power of an optical telescope operating at 457 nm (1 nm = 10^{-9} m) and a radio telescope operating at 1 cm, both having the same diameter of 200 mm. 2+3

(b) Draw a sketch of the Hertzsprung – Russell (H-R) diagram, labelling the axes carefully. Include in your sketch the approximate positions of the sun, the main sequence, white dwarfs, giants and the supergiants. 5

(c) Suppose a volume V of a universe contains material with an internal energy $U = \rho c^2 V$ (ρ is the density) and pressure P. Assume that, as the universe expands the energy of a comoving volume is conserved. Using thermodynamic argument, show that

$$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{P}{c^2}\right) = 0$$

where R is the scale factor and dots denote differentiation with respect to cosmic time. 5

(d) Argue from the first principles that in empty space, the form of radiative transfer equation is $\frac{dI_\nu}{ds}=0$, where I_ν is the specific intensity and 's' is the distance travelled. 5

(e) Obtain an expression for the free fall time T_{FF} of a spherical mass of gas which collapes without any outward support of pressure. 5

Answer ***any four*** questions. 10×4

3. (a) The Faintest object detectable with a large modern telescope in the sky has an apparent magnitude m = +29, whereas our sun has the apparent magnitude $m_{sun}= -26.81$. Compare their brightness, i.e. how much brighter will the sun appear than that Faintest object.

(b) Luminosity (L) of a star is defined as the total energy radiated by the star per unit time. Consider two stars of radii R_1 and R_2, surface temperatures T_1 and T_2 and absolute magnitudes M_1 and M_2, respectively. Using Stefan–Boltzmann law of radiation, show that

$$\frac{R_2^2\,T_2^4}{R_1^2\,T_1^4}=10^{0.4(M_1-M_2)}$$

(c) What do you mean by stellar parallax and how can we use this to measure astronomical distances? 4+3+3

4. (a) The ratio of brightness can be expressed in terms of absolute magnitude M, which for an astronomical object can be defined as its apparent magnitude if it were at a distance of 10pc from us. Using this idea, prove that the distance modulus of a star situated at 'r' parsecs from us is given by $m - M = 5\log_{10} r - 5$, where 'm' is the apparent magnitude. Hence relate absolute magnitude of a star with its luminosity.

(b) Assuming the initial statistical mechanical ideas of local thermodynamic equilibrium between two energy states, obtain the Saha Ionization equation for a stellar gas consisting of pure, partially ionized hydrogen. (3+3)+4

5. (a) Suppose that a star of total mass M and radius R has a density profile $\rho(r) = \rho_c(1 - r/R)$ where ρ_c is the central density

(i) Find $M(r)$.

(ii) Express the total mass M in terms of R and ρ_c.

(iii) Solve for the pressure profile $P(r)$, with the boundary condition $P(R) = 0$

(b) For a non-relativistic degenerate electron gas for which $p \propto \rho^{5/3}$ (p is pressure and ρ is mass density), use scaling arguments to show that the radius of a non relativistic white dwarf which is supported by electron degeneracy scales with its mass as $R \propto M^{-1/3}$ and $\rho \propto M^{-1/2}$. (1½+1½+3)+4

6. (a) Briefly discuss the evolution of stars, starting from the main sequence to creation of white dwarfs, neutron stars and black holes. Include estimates of the mass of stars on the main sequence that eventually produce each type of those remnants. What observational evidences are there for the existence of each type of remnant?

(b) The estimated lifetime of the sun on the main sequence is ~ 10^{10} years. Calculate the main sequence lifetime of a star which is ten times more massive than the sun.

(c) Write a short note on the Chandrasekhar mass limit. 6+2+2

7. (a) For a $K = 0$, $\Omega_{\wedge} = 1$ universe, suppose that at $t = 0$, there already existed a scale factor R_o. Show that for this case, the comoving paticle horizon r_H approaches a constant value c/H_oR_o and therefore galaxies beyond this would never be visible.

(b) Assuming the Robertson Walker metric to be of the form

$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$ and the two field equation of the form

$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}; \frac{\ddot{R}}{R} = \frac{-4\pi G}{3c^2}\times\left(\rho c^2 + 3P\right)$. Write down the expressions for Hubble's constant, decelaration parameter and density parameter from the above expressions. 5+5

8. (a) State the cosmological principle and discuss whether it is compatible with observations.

(b) The two Friedmann equations are given as

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

where R is the scale factor, ρ is the mass density, p is the pressure, k is a constant that specifies the geomerty of the universe, Λ is the cosmological constant, c is the speed of light and dots denote differentiation with respect to cosmic time.

Use these equations to derive the fluid equation in the form

$$\frac{d}{dt}\left(\rho R^3\right) + \frac{p}{c^2}\frac{d}{dt}\left(R^3\right) = 0$$

From the above equations, deduce how density (ρ) depends on the scale factor (R) in matter-dominated and radiation-dominated universe. 3+(4+3)

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