2018

PHYSICS – HONOURS

Paper : CC - 1

Full Marks : 50

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer **question no. 1** and **any four** questions from the rest.

1. Answer **any five** questions :   2×5

   (a) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

   (b) State the order and degree of the differential equation $\dfrac{d^4 y}{dx^4} + \left(\dfrac{dy}{dx}\right)^3 + x^2 y = 0$

   (c) For $u = e^x \cos y$, verify that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

   (d) Check whether the three vectors $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$ and $\hat{k} + \hat{i}$ are linearly independent.

   (e) Show that for any vector $\vec{r}$, $\vec{r}.d\vec{r} = rd(|\vec{r}|)$

   (f) If $\vec{a}$ is a constant vector, prove that $\vec{\nabla}(\vec{a}.\vec{r}) = \vec{a}$

   (g) Show that a Hermitian matrix remains Hermitian under a unitary transformation.

2. (a) Consider the function $f(x) = x\,|x|$. Sketch this function for both positive and negative values of x. Is the function continuous at $x = 0$? Check whether $f'(x)$ and $f''(x)$ exist at $x = 0$.

   (b) Find the first three terms in the Taylor expansion of $f(x) = \tan x$ about $x = \dfrac{\pi}{4}$.

   (c) Show that $df = -(y^2 + xy)dx + x^2 dy$ is not an exact differential but $\left(xy^2\right)^{-1} df$ is exact.

                                                                                              (2+1+2)+2+3

3. (a) Construct a second order differential equation of the form $y'' + ay' + by = 0$ (a, b are constants) for which the solutions are e^{3x} and e^{-2x}.

   (b) Use Lagrange multiplier to maximize $f(x, y) = x + y$ subject to the constraint $x^2 + y^2 = 1$.

   (c) Check whether the three vectors $\vec{A} = 2\hat{i} - \hat{j} - \hat{k}, \vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{C} = \hat{j} + \hat{k}$ are coplanar.

                                                                                              3+4+3

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4. (a) Show that $\vec{\nabla}.\left(\phi\vec{A}\right) = \vec{\nabla}\phi.\vec{A} + \phi\vec{\nabla}.\vec{A}$.

   (b) Find the values of λ, μ, ν so that the vector $\vec{F} = (x + \lambda y + 4z)\hat{i} + (2x - 3y + \mu z)\hat{j} + (\nu x - y + 2z)\hat{k}$ is conservative. Find also the scalar function $\phi(x, y, z)$ such that $\vec{F} = \vec{\nabla}\phi$.

   (c) Find a unit normal to the sphere $x^2 + y^2 + z^2 = 3$ at the point $(1, 1, -1)$. Evaluate $\oint_S \hat{r}.d\vec{S}$, where S is the surface of the given sphere. $\qquad$ 2+(2+3)+(2+1)

5. (a) State Stokes' theorem. Using this theorem evaluate $\oint \vec{A}.d\vec{r}$ around the boundary of the circle $x^2 + y^2 = 1$ in the anticlockwise direction for the vector field $\vec{A} = \left(x^2 - y^2\right)\hat{i} + 2x\hat{j} + 2\hat{k}$

   (b) Prove that $\oint u\vec{\nabla}v.d\vec{S} = \int u\nabla^2 v\, dV + \int \vec{\nabla}u.\vec{\nabla}v\, dV$, where the surface integral is over the surface enclosing the volume of integration.

   (c) Write down the Jacobian of (x, y, z) with respect to (r, θ, ϕ) where (x, y, z) and (r, θ, ϕ) are the Cartesian coordinates and the spherical polar coordinates respectively. $\qquad$ (2+3)+2+3

6. (a) Using the concept of Wronskian, show that the functions 1, x and $\sin x$ are linearly independent.

   (b) Find the eigenvalues and eigenvectors of the matrix $M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$. Are the two eigenvectors orthogonal?

   (c) If a matrix M satisfies $M^2 = 1$, what are its possible eigenvalues? $\qquad$ 3+(4+1)+2

7. (a) Show that Trace (AB) = Trace (BA) where A and B are two $n \times n$ matrices.

   (b) Show that the determinant of a unitary matrix is a complex number with modulus unity.

   (c) Solve the system of equations :
   $$\frac{dx}{dt} = x + y$$
   $$\frac{dy}{dt} = 3x - y$$
   with initial conditions $x(t = 0) = 0$, $y(t = 0) = 1$, by matrix method. $\qquad$ 3+3+4