2018

PHYSICS – HONOURS

Paper : CC-2

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** of the following : 2×5

(a) Show that for a body of variable mass m moving with momentum $\vec{p}$ and acted upon by an external force $\vec{F}$

$$\frac{d}{dt}(mT) = \vec{F}.\vec{p}$$

where T is the kinetic energy.

(b) For what kind of a rigid body will the angular momentum and angular velocity always be parallel?

(c) What do you mean by "Internal Bending Moment"— explain with a diagram.

(d) For a fluid at rest in a non-conservative force field, show that Pascal's law is not valid.

(e) Determine whether the following force is conservative or non-conservative :

$$\vec{F} = \left(2xy^2 - yz\right)\hat{i} + \left(2x^2y - xz\right)\hat{j} - xy\hat{k}$$

(f) A solid sphere and a solid cylinder, having the same mass and radius, start from same height at the same instant and roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.

2. (a) In two dimensions, a coordinate system is defined by

$$u = y - x$$
$$v = y + x$$

where (x, y) denotes the Cartesian system. If a particle of mass m starts at rest from the position $(x = 3, y = 3)$ under a force $\vec{F} = -k(x\hat{x} + y\hat{y})$, where k is a constant,

(i) Write down the accelerations a_u and a_v in the $u - v$ system and set up the equations of motion in this system.

(ii) Solve the equations of motion. Show the path of the particle indicating clearly the $\hat{u}$ and $\hat{v}$ unit vectors by drawing the $u - v$ coordinate system on top of the $x - y$ system.

(b) Show that the angular momentum of a system of particles splits up into the angular momentum about the centre of mass $\vec{L}_0$ and angular momentum of the centre of mass $\vec{L}_{cm}$ (assuming the mass of the body to be concentrated at the centre of mass). Hence show that

$$\vec{T}_0 = \frac{d\vec{L}_0}{dt} \quad \text{and} \quad \vec{T}_{cm} = \frac{d\vec{L}_{cm}}{dt}$$

Here $\vec{T}_0 = \sum_i \vec{r}_i' \times \vec{F}_i$ and $\vec{T}_{cm} = \vec{R} \times \sum_i \vec{F}_i$ where $\vec{r}_i'$ and $\vec{F}_i$ are the position with respect to the centre of mass and force on the ith particle. You may assume that $\vec{T} = \frac{d\vec{L}}{dt}$ where $\vec{T}$ and $\vec{L}$ are the total torque and total angular momentum of the system. [(2+1)+(2+1)]+4

3. (a) Show that the following definitions of a conservative force fields are equivalent.

$$\nabla \times \vec{F}(\vec{r}) = 0$$
$$\vec{F}(\vec{r}) = \nabla\phi(\vec{r})$$

where $\vec{F}(\vec{r})$ is the force field and $\phi(\vec{r})$ is a scalar function of position.

(b) Show that in a conservative force field the total energy is conserved.

(c) A solid cylinder is rotating about its symmetry axis. Measuring z from the bottom of the cylinder along the symmetry axis, the density of the body is given by $\rho(z) = \rho_0 \left(1 - \frac{z}{2l}\right)$ where ρ_0 is a constant and l is the length of cylinder. Calculate the moment of inertia of the cylinder about its axis, its radius being R. 4+3+3

4. (a) Prove that total energy of a particle of mass m acted upon by a central force is given by,

$$E = \frac{L^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where L is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$, r and θ being the polar co-ordinates.

(b) Derive Kepler's third law of planetary motion :

$$\frac{T^2}{a^3} = K \quad \text{constant}$$

where T is the time period, a is the semi-major axis of the elliptical orbit and K is a constant. Explain why, in the correct analysis, the constant K is not the same for all planets even if we assume that all planets in our solar system are moving only under the gravitational attraction of the sun.

(c) A uniform solid sphere of mass M is placed near an infinite plate whose mass per unit area is uniform and equal to σ. Prove that the sphere attracts the plate with a force of $2\pi GM\sigma$.

3+(3+1)+3

5. (a) Let S' be a reference frame which is rotating with respect to a frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$.

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refers to a time derivatives with respect to S and S' respectively.

(b) Assuming that a rigid reference frame fixed at the centre of the earth is inertial, set up the equations of motion with respect to a frame fixed on the surface of the earth for a particle of mass m moving under the gravitational force of the earth and other forces $\vec{F}_{other}$.

(c) Two particles of mass m_1 and m_2 move under a mutually interacting force acting along the line joining the position of two particles. Set up the equation of motion of the system and find an expression for reduced mass.

3+3+4

6. (a) Show that for a rigid body rotating with an angular velocity $\vec{\omega}$ and angular momentum $\vec{L}$, the kinetic energy T is given by

$$T = \frac{1}{2}\vec{\omega}.\vec{L}$$

Hence, show that

$$T = \frac{1}{2}\sum_{ij} I_{ij}\omega_i\omega_j$$

where I_{ij} are the components of the moment of inertia.

(b) Using Euler's equation of motion show that the kinetic energy is conserved for torque free motion. Demonstrate that in this case the angular velocity can only precess about its angular momentum vector at a fixed inclination.

(c) A string of length l with a mass m attached to it is wound around a horizontal flywheel having the shape of a solid cylinder of radius R and length L. As the mass decends under gravity the string unwinds and the mass acquires a velocity v at the instant when the string detaches from the flywheel. Find the mass of the flywheel. (Ignore friction in your analysis)

(2+1)+(2+1)+4

7. (a) Derive an expression for the energy stored in a wire of radius r and length l which is fixed at one end and twisted at other by an angle ϕ.

(b) Derive Poisuelle's equation for rate of fluid flow through a narrow tube.

(c) Water flows through a horizontal tube of length 20 cm and internal radius 0·081 cm. The pressure difference between the two ends of the tube is 20 cm of water. In 12 minutes 864 cm^3 of liquid flows through the tube. Calculate the coefficient of viscosity of water. 3+4+3