2019

ECONOMICS — HONOURS

Paper : CC - 4

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer ***any ten*** questions :

(a) What are level curves? Give an example of a level curve in economics. 1+1

(b) Consider the linearly homogeneous production function $Q = f(L, K)$. Using Euler's theorem, show that

(i) when marginal product of labour is zero, average product of capital equals marginal product of capital, and

(ii) when marginal product of capital is zero, average product of labour equals marginal product of labour 1+1

(c) State the duality theorem in the context of linear programming problems. 2

(d) Consider the function $f(x, y) = (x^2 - y^2)^2$. Is it a homothetic function? Justify your answer. 1+1

(e) For the following function, show that $f_{23} = f_{32}$ where f_{ij} denotes the second order partial derivative.

$$f(x_1, x_2, x_3) = \left(x_1^2 e^{3x_2 + x_1 x_3}\right) + \frac{2x_2^3}{x_1}.$$ 1+1

(f) Consider the Cobb–Douglas production function $y = 50 x_1^{1/3} x_2^{2/3}$. Show that the marginal product functions are homogeneous of degree zero. 1+1

(g) Define saddle point of a function $y = f(x_1, x_2, \ldots\ldots, x_n)$. 2

(h) Consider the expenditure function given by $e(p_1, p_2, u_0) = 2(u_0\, p_1\, p_2)^{1/2}$ where p_1 and p_2 are the prices of the two commodities and u_0 is the target utility level of the consumer. Find the Hicksian or compensated demand functions of the two commodities. 2

(i) Express the following matrix product as a quadratic form and check whether it is positive definite or negative definite. 2

$$\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

(j) Check whether the following function is concave or convex or neither : 2

$$z = 2x^2 - xy + y^2$$

(k) Comment on the nature of time path for the following equation : 2

$$y_t = -3\left(\frac{1}{4}\right)^t + 2$$

(l) Comment on the quasiconcavity / quasiconvexity of the function $z = x_1^2 + x_2^2$. 2

(m) What is 'value function' in a constrained optimization problem? Give an economic example of a value function. 1+1

(n) Use the implicit function theorem to show that $x^2y^3 + 3xy^2 + y = 22$ implies an explicitly defined function $y = f(x)$ at the point (1, 2) and find the value of the derivative $\frac{dy}{dx}$ at this point. 2

(o) Consider the utility function $u(x_1, x_2) = x_1^2 x_2$. Find the slope of the indifference curve corresponding to this utility function. 2

Group – B

Answer ***any three*** questions.

2. It is given that $\frac{dy}{dt} = (y - 3)(y - 5)$ where $y \geqslant 0$.

(a) Using a phase diagram, show that there are two possible equilibrium levels of y, one at $y = 3$ and the other at $y = 5$.

(b) Comment on the stability of the equilibria. 2+3

3. Let the utility functions of two individuals L and M be given by –

$$U_L = (X + a)^\alpha (Y + b)^\beta \quad \text{and} \quad U_M = [(X + a)^\alpha (Y + b)^\beta]^2$$

$a, b, \alpha, \beta > 0$. Do you think that the Indifference Curves (ICs) of the two individuals will have the same slope? Why? 1+4

4. State the 'Envelope Theorem'. Explain Roy's identity as an application of the Envelope Theorem. 2+3

5. If $u = Y^{2/3}L^{1/3}$ is the utility function of a person, where Y denotes wage income and L denotes leisure time enjoyed, find out her optimum leisure when wage rate is ₹ 100 per hour. 5

6. The profit function of a firm is given by

$$\pi = -R^2 - A^2 + 22R + 18A - 102$$

where A stands for advertisement expenditure and R stands for Research expenditure. Find out the optimum R and A for profit maximisation. Verify the second order condition. 3+2

Group – C

Answer ***any three*** questions.

7. Consider the following demand and supply functions :

$$Q_d = \alpha - \beta P + \sigma(dP/dt) \; ; \; Q_s = -\gamma + \delta P \; ; \; (\alpha, \beta, \gamma, \delta > 0)$$

(a) Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path $P(t)$.

(b) What is the intertemporal equilibrium price?

(c) What restriction on the parameter σ would ensure dynamic stability? 5+2+3

8. (a) A production function is given by $Q = A.L^{\frac{3}{4}} K^{\frac{1}{4}}$.

(i) What is the nature of returns to scale?

(ii) What is the share of K and L in the product, if each factor is paid a price equal to its marginal product? Show that the total product is exhausted.

(b) Which of the following functions are homothetic? (2+3)+(2½+2½)

(i) $e^{X^2Y} . e^{XY^2}$ (ii) $2\log X + 3\log Y$

9. "If $U(x_1, x_2)$ is homogeneous if degree 'r' in (x_1, x_2), then $V(p_1, p_2)$ is homogeneous of degree '$-r$' in (p_1, p_2), where V is the indirect utility function". Prove this result assuming $U = x_1 x_2$. 10

10. (a) Solve the following problem graphically :

Maximise : $R = q_1 + 2q_2$

Subject to : $q_1 + q_2 \leqslant 8$

$2q_1 + q_2 \leqslant 14$

$q_1, q_2 \geqslant 0$

(b) Consider the following problem :

Maximise $3x_1 + 4x_2$

Such that $x_1 + x_2 \leqslant 10$

$$2x_1 + 3x_2 \leqslant 18$$

$$x_1 \leqslant 8,\ x_2 \leqslant 6;\ x_1,\ x_2 \geqslant 0$$

If the optimal solution to its dual problem is $y_1^* = 0$, $y_2^* = 4/3$, $y_3^* = 1/3$, $y_4^* = 0$, find out the optimal solution to its primal problem and verify duality theorem. 5+5

11. Consider the following market model. Find the intertemporal equilibrium price and determine the nature of time path of price. Can you call the equilibrium a stable equilibrium? Illustrate your answer graphically.

$$Q_{dt} = 19 - 6P_t\ ;\quad Q_{st} = 6P_{t-1} - 5\ ;\quad Q_{dt} = Q_{st}$$ 2+3+2+3
