2022

PHYSICS — HONOURS

(Syllabus : 2018-2019 and 2019-2020)

Paper : CC-8

(Mathematical Physics III)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Prove that $\lim_{z\to 0}\left(\frac{z}{\bar{z}}\right)$ does not exist, where $\bar{z}$ is the complex conjugate of z.

(b) Expand $\frac{1}{z-2}$ in Taylor's series for $|z| < 2$, where z is complex.

(c) Find the residue of $f(z)=\frac{\sinh z}{z^5}$ at its singularity at $z = 0$.

(d) Write the Lagrangian of a simple pendulum and obtain its equation of motion using the Lagrangian.

Or, **(for 2018-2019 Syllabus only)**

Find the Fourier transform of $f(t)=e^{-i\omega t}$.

(e) Show using Variational principle that two Lagrangians whose difference is a total time derivative of a function of coordinates give same equation of motion.

Or, **(for 2018-2019 Syllabus only)**

For a random variable X, $\langle X\rangle = 2$ and $\langle X^2\rangle = 8$. Calculate the standard deviation.

(f) Show that the length of a moving rod is invariant in two inertial reference frames according to Galilean transformations.

(g) The mean lifetime of a muon at rest is 2.2 μs. Calculate the average distance that it will travel in vacuum before decay, if it starts moving with velocity 0.9c.

Please Turn Over

2. (a) (i) Prove that $u = 2x(1 - y)$ is harmonic.

(ii) Find a function v such that $f(z) = u + iv$ is analytic.

(iii) Express $f(z)$ in terms of z.

(b) Prove that the real and imaginary parts of an analytic function of a complex variable when expressed in polar form satisfy the Laplace's equation in polar form given by

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}=0.$$

(c) Locate and name all the singularities of

$$f(z)=\frac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$$

(1+2+2)+3+2

3. (a) Find the Laurent series expansion of the function $f(z)=\dfrac{1}{z^2(1-z)}$ in the domain $0 < |z| < 1$.

(b) Evaluate the following integrals :

(i) $\displaystyle\oint_C \frac{e^{2z}}{(z+1)^3}dz \quad C:|z|=2$

(ii) $\displaystyle\int_0^\infty \frac{\cos ax}{x^2+1} \quad a<0.$

3+(3+4)

4. (a) Space-time coordinates of a pair of events in an inertial frame S are $A\left(\frac{a}{c}, a, 0, 0\right)$ and $B\left(\frac{a}{2c}, 2a, 0, 0\right)$. Find the separation of the two events in an inertial frame S' in which two events are simultaneous. Also find the speed of S' with respect to S. Use the metric $(1, -1, -1, -1)$.

(b) An inertial frame S' is moving at a speed $c/2$ away from another inertial frame S along common $x-x'$ axis. As observed from S', a particle is moving with speed $c/2$ in y' direction. Find the speed of the particle as seen from S.

(c) Consider three inertial frames S, S' and S''. S' is moving with respect to S along the common x–x' axis with a velocity v. S'' is moving with respect to S′ along common x'–x'' axis with velocity v'. If the velocity of S'' with respect to S is v'', then show that $\gamma'' = \gamma\gamma'\left(1+\beta\beta'\right)$ where $\gamma=\dfrac{1}{\sqrt{1-v^2/c^2}}$ and similarly for γ' and γ'', also, $\beta=\dfrac{v}{c}$ and $\beta'=\dfrac{v'}{c}$

(2+1)+4+3

5. (a) Particle of mass M initially at rest, decays into two pieces each of mass m. Find out the speed of each particle.

(b) A particle of mass m whose total energy is twice its rest mass energy collides with an identical particle at rest. After collision, they stick together. Find the mass of the resulting composite particle. What is its velocity?

(c) Show that under Lorentz transformation $p^{\mu}p_{\mu}$ is invariant where p^{μ} is the 4-momentum of the particle. Use $(1, -1, -1, -1)$ metric. 3+(2+2)+3

6. (a) Using Variational principle find the Curve of shortest distance between two points in a plane.

(b) Consider the Lagrangian

$$L = \frac{1}{2}MV^2 + q\,\vec{V}.\vec{A},$$

where q is a constant scalar and $\vec{A}$ is a constant vector.

(i) Find the generalized momentum.

(ii) Find the Hamiltonian.

(c) A particle of mass m is constrained to move on a vertical circular loop of radius R. The gravity acts downwards. Construct the Lagrangian of the system. 3+(2+2)+3

***Or,* (for 2018-2019 Syllabus only)**

(a) Find the probability density function $f(x)$ for the position of a particle executing SHM on $(-a, a)$ along the x-axis.

(b) Let X be a random variable having a normalized density function

$$f(x) = \begin{cases} Ce^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) the value of $|C|$ where C is a constant.

(ii) the variance σ^2.

(iii) the probability $P(1 \leq x \leq 2)$. 3+(2+3+2)

7. A particle of mass m is thrown with a velocity u making an angle α with the x-axis. The gravity acts downwards. Consider the motion in X-Y plane.

(a) Construct the Lagrangian.

(b) Find the equation of motion.

(c) Find if there is any cyclic coordinate.

(d) Construct the Hamiltonian.

(e) Using Hamilton's equation of motion, Show that $\frac{dH}{dt} = 0$. 2+2+1+2+3

2022

PHYSICS — HONOURS

(Syllabus : 2019-2020)

Paper : CC-9

[Analog Electronics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) State and explain Thevenin's theorem.

(b) Can barrier potential be measured by a voltmeter?

(c) Why is the emitter current always greater than the collector current in normal biasing BJT?

(d) Mention the utilities of emitter bypass capacitor and coupling capacitor from collector in self biased configuration of BJT.

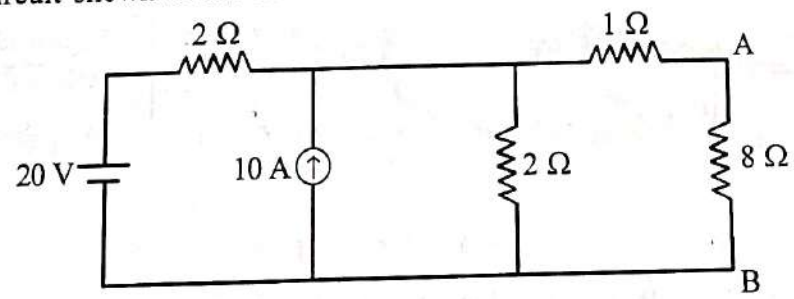
(e) What are the advantages of negative feedback in voltage amplifier?

(f) Draw the circuit diagram of a bridge rectifier.

(g) What is slew rate of an OPAMP?

2. (a) State and explain Norton's theorem.

(b) Solve the circuit shown below for the current in the branch AB using Norton's theorem.



(c) State Maximum Power transfer theorem. (2+1)+5+2

3. (a) Find the expression for the ripple factor of a half-wave rectifier.

(b) Explain with a circuit diagram, the use of Zener diode as a reference diode.

(c) Explain the principle of operation of an LED. Why is silicon not preferred as an LED material? 3+4+(2+1)

Please Turn Over

4. (a) What are the factors that affect the bias stability of a transistor? Define stability factor with respect to change of any one of them.

(b) A transistor is operating in CE configuration. A 560 Ω resistor is joined between the collector and power supply and a voltage drop of 0.6V occurs across it. If $\alpha = 0.97$, calculate the base current.

(c) Using *h* parameters, obtain an expression for input resistance of a CE amplifier. (2+1)+4+3

5. (a) What is a load line? Define Q-point with respect to load line.

(b) State the differences between depletion type and enhancement type MOSFET. Draw $I_D - V_{GS}$ transfer characteristics of depletion type MOSFET.

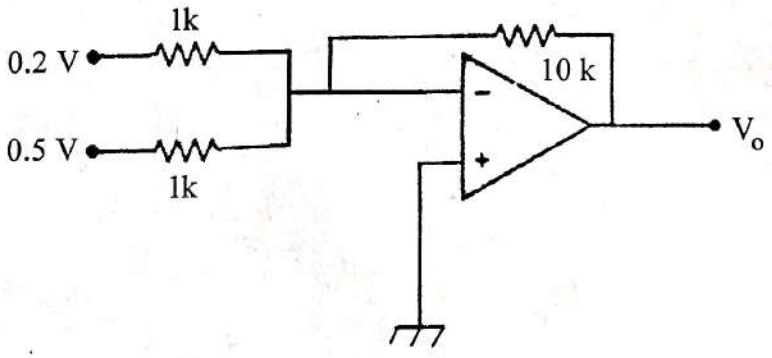
(c) When V_{GS} of a JFET changes from –3.1 V to –3 V, the drain current changes from 1 mA to 1.3 mA. What is the value of transconductance?

(d) What are the different ways of sampling the output signal in a feedback amplifier? Name the four feedback topologies. (1+1)+(2+2)+2+2

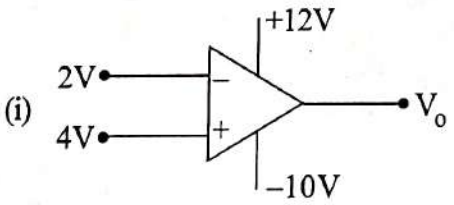
6. (a) Calculate the voltage gain of an inverting OPAMP.

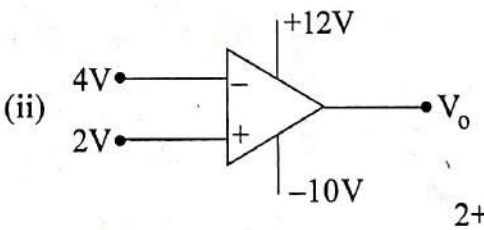
(b) What do you mean by CMRR? Deduce the expression for CMRR of OPAMP.

(c) Determine the output voltage of the following circuit :



(d) Find out the output voltage of the following two circuits :





2+(1+2)+3+(1+1)

7. (a) Draw the circuit digram of a series regulated power supply with two transistors taking reference voltage from a reverse biased Zener diode.

(b) Draw the circuit diagram of a Wien Bridge Oscillator. Prove that the gain of the amplifier used in Wien Bridge Oscillator must be greater than 3 for sustained oscillation.

(c) Draw the circuit diagram of a monostable multivibrator and explain briefly its operation. 3+4+3

2022

PHYSICS — HONOURS

(2019–2020 Syllabus)

Paper : CC-10

(Quantum Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** 1 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Find the degeneracy of an energy level with principal quantum number n for a hydrogen atom.

(b) Can Lithium ($Z = 3$) give rise to Normal Zeeman effect? Explain.

(c) Show that $[L^2, L_\pm] = 0$ when $L_\pm = L_x \pm L_y$. The symbols have their usual meanings.

(d) Find the eigenvalues and eigenfunctions of angular momentum operator $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$.

(e) Define degeneracy. What is meant by degree of degeneracy?

(f) What are L-S and J-J coupling?

(g) What is Hund's rule?

2. (a) A particle is described by the following wave function (at $t = 0$) :

$$\psi(x,0) = \sqrt{\frac{1}{6}}\psi_1(x) + \frac{i}{\sqrt{2}}\psi_2(x) + \frac{1}{\sqrt{3}}\psi_3(x)$$

ψ_1, ψ_2, ψ_3 are the first three energy eigenstates of the harmonic oscillator. How will such state evolve in time? Find the average energy of the particle.

(b) Obtain the expectation values of $\hat{x}$, $\hat{p}$ and x^2 for the ground state of simple harmonic oscillator.

(2+2)+(2+2+2)

3. For a linear harmonic operator with $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, $\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}}[\hat{p} - im\omega\hat{x}]$

and $a^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}[\hat{p} + im\omega\hat{x}]$

Please Turn Over

(a) Find $[\hat{H},\hat{a}],[\hat{H},\hat{a}^\dagger],[\hat{H},\hat{a}\hat{a}^\dagger]$

(b) Show that $\hat{H}=\left(\hat{a}\hat{a}^\dagger-\frac{1}{2}\right)\hbar\omega$

(c) Show that $\hat{H}\,\hat{a}\psi_n(x)=\left(n-\frac{1}{2}\right)\hbar\omega\psi_n(x)$

If energy corresponding to a particular state $\psi_n(x)$ is $\left(n+\frac{1}{2}\right)\hbar\omega$. 4+3+3

4. The initial $(t=0)$ wave function of a free particle is described by the Gaussian wave packet $\psi_0(x)=Ae^{-\alpha x^2}$, where A and $\alpha(\alpha>0)$ are constants.

(a) Normalize the wave function $\psi_0(x)$.

(b) Find the wave function $\psi(x,t)$ of the free particle at a later time t.

(c) Calculate the probability density and hence show that the wave packet of the free particle broadens spatially with time.

(d) Plot the probability density at time $t=0$ and time $t>0$ with x. 2+4+3+1

5. (a) Express the operators L^2 and L_z in spherical polar coordinate system. Hence verify that $\psi(\theta,\phi)=\left(\frac{15}{32\pi}\right)^{\frac{1}{2}}\sin^2\theta e^{2i\phi}$ is an eigenfunction of both L^2 and L_z. Find the eigenvalues.

(b) A spin $\frac{1}{2}$ particle is in a state $|\alpha\rangle=\frac{1}{\sqrt{30}}\begin{pmatrix}2-i\\5\end{pmatrix}$. If S_z is measured, then find the probabilities of getting eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$. (4+2+1)+3

6. (a) Consider two non-interacting identical particles of mass m in an infinite square well. The single particle state is given as

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right),$$

when $E_n=n^2K$, a is the width of the well from $x=0$ to $x=a$ and $K=\frac{\pi^2\hbar^2}{2ma^2}$.

(i) Find the ground state normalized wave functions and energies if the particles are identical bosons and identical fermions.

(ii) Find the normalized wave function for first excited state and the corresponding energy if the particles are identical bosons.

(b) (i) In Stern-Gerlach experiment, why is it necessary to use beam of neutral atoms and not of ions?

(ii) In this experiment, a beam of neutral nickel atom splits into nine components. What is the angular momentum of a nickel atom in its ground state? (4+2)+(2+2)

7. (a) Using first-order perturbation theory, calculate the shift of energy with respect to the Bohr energy of hydrogen atom due to spin-orbit interaction term.

(b) Hence, derive the complete fine-structure formula for hydrogen atom if the relativistic correction to kinetic energy is $E_r^{\,l} = -\frac{(E_n)^2}{2mc^2}\left(\frac{4n}{l+\frac{1}{2}} - 3\right)$, where symbols have their usual meanings.

(c) Write down the electronic structure of Carbon Atom. Hence, express the ground state of the atom with the help of Term symbol. 4+3+3

2022

PHYSICS — HONOURS

(Syllabus : 2019-20)

Paper : SEC-B-1

[Arduino]

Full Marks : 20

Time : 30 minutes

Answer ***any ten*** questions each carrying 2 marks.

1. The programme written in Arduino IDE is known as
   (a) Module
   (b) Link
   (c) Script
   (d) Sketch

2. In the following programme what will be the output in serial monitor if 3.7 volt is applied to pin A0?

```
void setup( ) {
serial.begin(9600);
}
void loop( ) {
int sensorValue = analogRead(A0)
serial.println(sensorValue);
}
```

   (a) 353
   (b) 512
   (c) 613
   (d) 757

3. What will be the output of the following code?

```
#define X 10;
void setup( ) {
    X = 0;
Serial.begin(9600);
Serial.print(X);
}
void loop( ) {
//Do nothing
}
```

(a) 0

(b) 10

(c) X

(d) Error

4. What are the two modes that the `pinMode( )` command sets for a particular pin?

(a) HIGH and LOW

(b) DIGITAL and ANALOG

(c) READ and WRITE

(d) INPUT and OUTPUT

5. What type of signal does the `analogWrite( )` function output?

(a) Analog signal of constant amplitude

(b) Pulse width modulated signal

(c) Amplitude modulated signal

(d) Frequency modulated signal.

6. What is the correct order of execution process of an Arduino code?

(a) Editor → Preprocessor → Compiler

(b) Editor → Compiler → Preprocessor

(c) Preprocessor → Compiler → Editor

(d) Preprocessor → Editor → Compiler

7. What is the resolution of `analogRead( )`?
   (a) 4.9 mV
   (b) 4 mV
   (c) 5 mV
   (d) 7 mV

8. Number of digital pins in Arduino UNO are
   (a) 11
   (b) 12
   (c) 15
   (d) 13

9. The basic function of ADC is to
   (a) Convert Analog to Digital signal
   (b) Convert Digital to Analog signal
   (c) Convert Digital pin to Analog
   (d) Convert Analog pin to Digital

10. The clock speed of the Arduino UNO board is
   (a) 32 MHz
   (b) 16 MHz
   (c) 1 MHz
   (d) 5 MHz

11. TX pin represents _____ in the Arduino Board.
   (a) Transmitter
   (b) Receiver
   (c) Reset
   (d) Export

12. IC LM35 is a IC of
   (a) Temperature controller
   (b) Temperature sensor
   (c) Voltage controller
   (d) Current controller