2022

PHYSICS — HONOURS

Paper : CC-5

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Explain what is the physical meaning of the constant term in a Fourier series expansion.

(b) Using the Generating function for the Legendre polynomials

$$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n,$$

show that $p_n(-1) = (-1)^n$, where n is a positive integer.

(c) Discuss about the singularities of the following equations :

(i) $\dfrac{d^2y}{dx^2} - \dfrac{6}{x^2}\, y = 0$.

(ii) $\dfrac{d^2y}{dx^2} - \dfrac{6}{x^3}\, y = 0$.

(d) In an α-particle counting experiment, the number of α-particles is recorded in each minute for two hours. The total number of particles is 500. In how many 1-minute intervals do you expect no particle?

Or, [Syllabus 2018-19]

Which symmetry of the Lagrangian does the conservation of Hamiltonian (Energy) comes from? Justify.

(e) Obtain the Parseval identity for a Fourier series.

Or, [Syllabus 2018-19]

Show that if one adds, to the Lagrangian of a system, a total time derivative of a function of co-ordinate and time only, the equation of motion remains invariant.

(f) Show that $\Gamma(z + 1) = z\,\Gamma(z)$ for any z.

(g) Show that $B(p, q) = B(q, p)$, where $B(q, p)$ is the beta function.

2. (a) Find the Fourier transform $g(k)$ of the function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-x^2/2\sigma^2\right]$$

and plot both $f(x)$ and $g(k)$.

(b) What is the physical meaning of σ? Find the corresponding quantity of $g(k)$ and show how they are related.

(c) When $\sigma \to 0$ limit is taken what will happen to $f(x)$ and to $g(k)$? 5+3+2

Or, [Syllabus 2018-19]

(a) Find the shortest distance between two nearby points in 2-dimensional Euclidian Space using variational principle.

(b) Two bodies of mass m_1 and m_2 are hanging under gravity from the two ends of an inextensible string of length l which goes over a frictionless, massless pulley. Is it a constrained system? Find the Lagrangian and equations of motion of the masses. What is the force of constraint here? 4+(1+2+2+1)

3. (a) Evaluate $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ using Beta and Gamma functions.

(b) Show that $B(n, n) = B(n, \frac{1}{2})/2^{2n-1}$

(c) Show that $\Gamma(2n) = \frac{1}{\sqrt{\pi}} 2^{2n-1} \Gamma(n) \Gamma\left(n + \frac{1}{2}\right)$

(d) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 2+3+2+3

4. (a) Find the Fourier Series for the periodic function $f(x) = e^x$, $-\pi \le x < \pi$, with a period 2π.

(b) If $f(x) = f(-x)$ and $g(x) = -g(-x)$, show which terms should be present in the Fourier expansion of $f(x)$ and $g(x)$, with period $-1 \le x < 1$.

(c) Find the Fourier series expansion for $f(x) = \cos^2 x$. 5+3+2

5. (a) Prove that the Legendre polynomials of different orders are orthogonal.

(b) Using the expression for Bessel's function

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(m+r+1)} \left(\frac{x}{2}\right)^{n+2r},$$

Show that,

(i) $J_{m+1}(x) + J_{m-1}(x) = \frac{2m}{x} J_m(x)$

(ii) $J_{m-1}(x) - J_{m+1}(x) = 2J'_m(x)$ 4+(3+3)

6. (a) How does the Fourier transform $g(k)$ of a function $f(x)$ change under the translation $x \to x + a$, where a is some constant?

(b) If $g(k)$ is the Fourier transform of $f(x)$, what is the Fourier transform of $f^*(x)$?

(c) Show that in a certain limit Binomial distribution can be converted to Gaussian (Normal) distribution.

(d) Show that Fourier transform of $f(x) = 1$ has the properties of Dirac's δ-function by integrating from $-L$ to $+L$ and then taking $L \to \infty$ limit. 2+2+3+3

Or, [Syllabus 2018-19]

(a) Show that if the Lagrangian is invariant under rigid rotation, then angular momentum of the system is conserved.

(b) Let us consider the Lagrangian in polar coordinate $L = \frac{1}{2} m\left(\dot{r}^2 + r^2\,\dot{\theta}^2\right) - V(r)$.

Find the equations of motions. Is there any cyclic coordinate? What are the conserved quantities and why? 5+(2+1+2)

7. (a) A bar 10 cm long with insulated sides is initially at 100°C. Starting at $t = 0$, its ends are held at 0°C. Find the temperature distribution in the bar at time t. Use the Heat flow equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$$

where $u(x,t)$ is the temperature, α is a constant.

(b) Consider the vibration of a circular membrane obeying

$$\nabla^2 \Psi = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

where $\Psi = \Psi(r, \theta, t)$.

Find the solution of the equation by separation of variables method. What will be the boundary conditions needed here? Take the radius of the membrane to be R. The circumference is held fixed. 4+6

2022

PHYSICS — HONOURS

Paper : CC-6

(Thermal Physics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

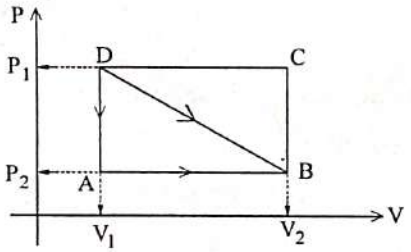
Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Distinguish between extensive and intensive variables.

(b) The mean free path of a gas is 5.0 cm. Among 100 free paths of those molecules, how many are between 4.9 cm. and 5.1 cm?

(c) Using the indicator diagram shown below, show that the work done is not a state function.



(d) Explain the concept of temperature on the basis of Zeroth law of thermodynamics.

(e) What is the reason for considering Quasi-static process in the context of thermodynamics?

(f) Show that Clausius's theorem leads to the concept of entropy as a state function.

(g) State Gibbs phase rule. Explain the rule with an example.

2. (a) Write down the assumptions used in the derivation of Maxwell's speed distribution law.

(b) According to Maxwell's speed distribution law, the number of molecules per unit volume with speed between v and v + dv is given by $n(v)dv = na^3 e^{-b\left(v_x^2+v_y^2+v_z^2\right)} dv_x dv_y dv_z$ where symbols have their usual meaning. Calculate the constant 'a' in terms of the constant 'b'.

(c) Show that Maxwell's speed distribution law is normalized.

(d) Calculate the average of x-component of velocity of a Maxwellian gas. 2+3+3+2

Please Turn Over

3. (a) Equation of state of a non-ideal gas is given by $P(V-b)=RT\exp\left(-\frac{a}{RVT}\right)$ [a, b are constants].

Show that the above equation reduces to the ideal gas equation (i) as $V \to \alpha$ and (ii) if 'a' and 'b' are small.

(b) What is Brownian motion?

(c) Obtain the expression for the mean free path of a molecule of an ideal gas as a function of its molecular diameter. (2+2)+2+4

4. (a) What do you mean by internal energy of a thermodynamic system? What are the limitations of the first law of thermodynamics?

(b) What is adiabatic lapse rate? Find an expression for it.

(c) A certain gas has equation of state $P=\frac{\alpha N^2 T}{V^2}$, where P is the pressure, N is the number of moles, V is the volume, T is the temperature and α is a constant. One mole of the gas undergoes expansion from volume V to $2V$ at a constant temperature T. If the change in energy in the isothermal expansion is $\beta\frac{\alpha T}{V}$, find the value of β. (1+2)+(1+3)+3

5. (a) "The perpetual motion machine of 2nd kind is impossible to construct." — Justify this statement.

(b) Starting from 2nd law of thermodynamics show that for a mechanically isolated system at constant temperature, the Helmholtz free energy never increases.

(c) Derive Clausius-Clapeyron equation from TdS equation.

(d) Write down the characteristics of second-order phase transition with a suitable example. 2+3+3+2

6. (a) What is entropy? State its properties.

(b) Entropy of an ideal gas with N number of molecules in a volume V is given by

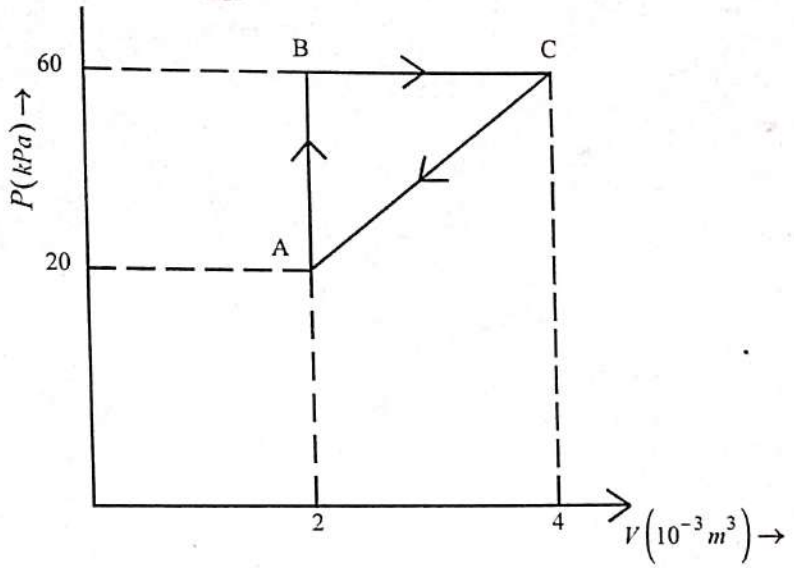
$$S=Nk_B \ln\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3Nk_B}{2},$$

where m is the mass of one molecule, E is the energy, h is Planck's constant and k_B is the Boltzmann constant.

Show that this expression for entropy does not satisfy extensive property of entropy and leads to Gibbs paradox.

(c) How is this paradox removed?

(d)



In the cycle ABC, heat is added to a thermodynamic system in the process AB and BC are 400 J and 100 J respectively. Heat rejected during the process CA is 460 J. Find its efficiency.

(1+2)+3+2+2

7. (a) Distinguish between free expansion and Joule-Thomson expansion.

(b) What is Joule-Thomson effect? Show that Joule-Thomson coefficient of a real gas is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right],$$

where symbols have their usual meaning.

(c) Calculate the rate of heat flow through a composite slab of widths 2 cm and 0.8 cm with thermal conductivities of 0.043 $Wm^{-1}K^{-1}$ and 0.11 $Wm^{-1}K^{-1}$ respectively. The cross-sectional area of the composite slab is 26 cm^2 and the temperature difference between the two faces of the slab is 20°C.

2+(2+3)+3

2022

PHYSICS — HONOURS

Paper : CC-7

(Syllabus : 2019-2020)

[Modern Physics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) What is the importance of the Davisson–Germer experiment?

(b) Find the eigenstate of $i\frac{d}{dx}$.

(c) Is $\Psi(x,t) = c_1\psi_1(x)e^{-iE_1t/\hbar} + c_2\psi_2(x)e^{-iE_2t/\hbar}$ a stationary state?

(d) State the Superposition Principle as understood in quantum mechanics.

(e) Calculate binding energy for ${}^{16}_{8}\mathrm{O}$. Given that

$M\left({}^{1}_{1}H\right) = 1.007825\,\mathrm{u}$, $\mathrm{M}\left({}^{1}_{0}n\right) = 1.008665\,\mathrm{u}$, $\mathrm{M}\left({}^{16}_{8}O\right) = 15.994915\,\mathrm{u}$ and $1\,\mathrm{u} = 931.5\,\mathrm{MeV}$.

(f) In stable nuclei, why are neutrons in excess of protons?

(g) What is the difference between temporal coherence and spatial coherence?

2. (a) Assume we have a two-slit experiment in which the screen is replaced by a large planar array of detectors for charged particles, and normal to this and towards this is sent electrons from an electron gun. In the path between the electron gun and the detector array are two narrow slits, the line joining them being parallel to the 'screen'. What kind of pattern is detected on the detector array ("screen") when (i) both slits are open (ii) either slit is closed? Justify your answers. Suppose the electron gun emits one electron at a time. To determine which slit it passes through, we place a small detector near one of the slits. What do we observe on the 'screen'? Explain.

(b) A photon of wavelength λ is scattered by an electron, initially at rest. Show that the change in the wavelength of photon is given by $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$, where θ is the angle of scattering of the photon.

Please Turn Over

(c) How do you interpret the de Broglie wave in quantum mechanics? Calculate the ratio of de Broglie wavelength of an electron and an α-particle accelerated by the same potential difference.

3+4+(2+1)

3. (a) Show that the operator $\frac{d}{dx}$ is not Hermitian.

(b) Using the equations $\left[\hat{x}_i, \hat{p}_j\right] = i\hbar\delta_{ij}$ $(i, j = 1, 2, 3)$, show that for $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$, $\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$, $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$, we must have $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$.

(c) For the first excited state of the harmonic oscillator the wave function is $x\,exp\left[-\frac{m\omega}{2h}x^2\right]$.

Show that the value of the uncertainty product $\Delta x \Delta p$ is $\frac{3}{2}h$. 3+4+3

4. (a) Consider 1D linear harmonic oscillator with potential

$$V(x) = \frac{1}{2}m\omega^2 x^2.$$

The ground state wave function is given by

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}, \text{ where } \alpha = \frac{m\omega}{\hbar}.$$

Calculate average kinetic energy and average potential energy and hence show that ground state energy is given by

$$E_0 = \frac{1}{2}\hbar\omega.$$

(b) Derive an expression for the time derivative of the expectation value of the momentum operator for a particle moving in one-dimension, that is, for

$$\frac{d\langle\hat{p}_x\rangle}{dt}.$$

(c) For a finite square well of depth $V(x) = -V_0$ in one-dimension, given a plane wave $Ae^{i(p_x x - Et)/\hbar}$ incident on it, discuss qualitatively what happens afterwards for various values of E relative to $|V_0|$.

5+3+2

5. (a) Is the nuclear force between two protons different from that between a proton and a neutron? Discuss.

(b) Show that in β decay ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^{-} + \bar{\nu}_e$, the kinetic energy of recoil nucleus Y is given by

$$E_Y = \left[\frac{Q + 2m_0c^2}{2M_Yc^2}\right] T_{max}$$

where Q = β disintegration energy, T_{max} = maximum kinetic energy of β particles. Assume that the motion of the recoil nucleus is non-relativistic.

(c) Discuss briefly the phenomenon of pair production. Does electron-positron pair creation by a gamma ray photon in the vicinity of a nucleus fall within the realm of non-relativistic quantum mechanics? Justify your answer. 2+4+(3+1)

6. (a) Describe the fission of a nucleus based on the liquid drop model.

(b) ^{235}U is fissile with slow neutron but ^{238}U is not, why?

(c) Write short notes on :

(i) Primary source of energy of the sun

(ii) Carbon-nitrogen-oxygen cycle (also known as the Bethe-Weiszäcker cycle), the principal source of energy in stars exceeding 1.3 solar masses. 3+3+(2+2)

7. (a) What is 'population inversion'?

(b) List the possible sources of 'line broadening'.

(c) Establish a relation between Einstein's A, B coefficients and hence comment on the incoherency observed in ordinary light. 3+3+4

2022

PHYSICS — HONOURS

Paper : SEC-A-1

[Syllabus : 2019-2020]

(Scientific Writing)

Full Marks : 20

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any ten*** questions. 2×10

1. Which of the following commands for font typesetting is not valid in LaTeX?

   (a) \textsc (b) \textit

   (c) \textbf (d) \textmr

2. In a \begin{tabular} environment, what is the correct code for producing a table with 3 columns— first 2 are left-aligned and the third is right-aligned with no vertical lines among them?

   (a) {| l | l | r} (b) {l l r}

   (c) {| \l | \l | \r} (d) {\l \l \r}

3. What is the output of the LaTeX command $ \frac{5}{\frac{3}{2} + 1} $?

   (a) $\dfrac{5}{\frac{3}{2}+1}$ (b) $\dfrac{5}{\frac{3}{2}}+1$

   (c) $\dfrac{\frac{3}{2}+1}{5}$ (d) $5\dfrac{3}{2}+1$

4. In a math mode, what is the correct command to write $\sqrt[x]{y^{1/3}}$ in LaTeX?

   (a) \sqrt {y}^{1/3}_x (b) \xsqrt{y^{1/3}}

   (c) \sqrt[x]{y^{1/3}} (d) \sqrt{x}{y^{1/3}}

5. How to write $\zeta(2)$ inside math mode in LaTeX?

   (a) \zeta(2) (b) \zeta{2}

   (c) \xi(2) (d) \xi{2}

Please Turn Over

6. What is the correct output of the LaTeX command S\prod_{i=1}^ni = n! S?

(a) $\Pi_{i=1}^{ni} = n!$ (b) $\Pi_{i=1} ni = n!$

(c) $\Pi_{i=1}^{n} i = n!$ (d) $\Pi_{i=1} n^i = n!$

7. What is the correct LaTeX command to write ***examinee*** inside a text?

(a) \textit{\textbf{examinee}} (b) {\textit \textbf examinee}

(c) {\it {\bf examinee}} (d) {\it \bf examinee}.

8. The symbol $\Rightarrow$ is written inside math mode in LaTeX as

(a) \Rightarrow (b) \rightarrow

(c) \Doublerightarrow (d) \doublerightarrow.

9. What is the use of % symbol inside a text in LaTeX?

(a) To calculate percentage

(b) To convert into percentage

(c) To make rest of the line out of compilation

(d) To convert into math mode.

10. A block \[\] makes the content inside it to

(a) surround by square bracket (b) keep the section out of compilation

(c) produce matrix (d) present equation in display mode.

11. A paragraph is started with indentation. What is the LaTeX command to make the indentation off?

(a) \indentoff (b) \noindent

(c) \indent = 0 (d) \zeroindent

12. What is the LaTeX command to scale up a figure by 30% of its original size inside an appropriate figure environment?

(a) \includegraphics [scale = 30%] { } (b) \includegraphics [scale = 1.3] { }

(c) \includegraphics [scale = 0.3] { } (d) \includegraphics [scale = 130] { }