14B

2022

PHYSICS — HONOURS

Paper : CC-11

(Syllabus : 2019-2020)

[Electromagnetic Theory]

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Find the velocity of light in a medium for which relative permittivity is 3 and permeability is half of the value of permittivity.

(b) The intensity of sunlight hitting the earth is about 1300 Watt/m^2. Find the amplitude of the electric field.

(c) Show that the frequency of the wave remains unchanged upon refraction, when an electromagnetic wave is incident on the plane interface between two different media.

(d) Calculate the reflection coefficient for an electromagnetic wave of frequency 10 GHz when it is incident normally on a metal surface of conductivity 6×10^7 $(\Omega\text{m})^{-1}$. Take $\epsilon = \epsilon_0$ and $\mu = \mu_0$.

(e) In a current free region $B_x = ax + bz$ and $B_y = ax + cy$. Find a possible form for B_z assuming all currents are outside.

(f) Define specific rotation.

(g) Define the state of polarization of the wave presented by the following set of equations $E_x = E_o \sin(kz - \omega t)$ and $E_y = E_o \cos(kz - \omega t)$.

2. (a) Show how Maxwell's equations in free space imply local conservation of charge. Show that this implies Kirchoff's second law.

(b) Starting from Maxwell's equation derive the following equations :

$$\left(\nabla^2\vec{A} - \mu_o\epsilon_o\frac{\partial^2\vec{A}}{\partial t^2}\right) - \bar{\nabla}L = -\mu_o\vec{J} \text{ and } \left(\nabla^2\Phi - \mu_o\epsilon_o\frac{\partial^2\Phi}{\partial t^2}\right) + \frac{\partial L}{\partial t} = -\frac{\rho}{\epsilon_o},$$

where symbols have their usual meanings. Comment on the term L. Hence derive modified Poisson equation. (2+2)+(3+2+1)

3. (a) A long coaxial cable of length l consists of an inner conductor of radius a and an outer conductor of radius b. It is maintained at a potential difference V. The current flows down the inner cylinder and back along the outer cylinder. If λ be the charge per unit length of the inner cylinder, find the electromagnetic energy stored in the cable.

(b) Show that the momentum density stored in electromagnetic field is given by $\vec{S}/c^2$ in vacuum; where $\vec{S}$ is the Poynting vector.

(c) Show that if a monochromatic linearly polarized plane wave is moving in an isotropic non-conducting medium, the time average of its energy density is distributed equally between the magnetic and electric fields. 3+4+3

4. (a) Determine the expression of the phase angle in a conductor lagged by the magnetic field behind the electric field. Determine the angle for a ideal conductor.

(b) Define a good and a bad conductor from the point of view of the frequency of the incident electromagnetic wave. A metal has conductivity of 7.2×10^{7} $(\Omega\,\text{m})^{-1}$. Obtain the skin depth in that metal at frequency of 1 kHz. Why are metals opaque? (3+2)+(2+2+1)

5. (a) A current sheet of surface density $\vec{K} = -10\hat{y}\ A/m$ is located in the plane $z = 0$. The permiabilities of the meterial for $z > 0$ and $z < 0$ are $5\mu_o$ and $2\mu_o$ respectively. The magnetic field in the region $z > 0$ is $\vec{H}_> = 15\hat{x} - 8\hat{z}\ A/m$. Find $\vec{B}_>$, $\vec{H}_<$ and $\vec{B}_<$, where

$\vec{H}_>, \vec{B}_>$ stand for fields at $z > 0$, and

$\vec{H}_<, \vec{B}_<$ stand for fields at $z < 0$.

(b) A plane electromagnetic wave falls obliquely on the interface between two simple dielectrics. The electric field vector is parallel to the plane of incidence. Obtain expressions for the transmission coefficient.

(c) A plane EM wave travelling in free space is incident normally on a dielectric material with dielectric constant of 2.5. If there is no absorption by the material, find the reflectivity and transmittivity. 3+4+3

6. (a) Explain how to detect the following waves :

(i) circularly polarized, (ii) elliptically polarized.

(b) What is Babinet's compensator? Write the advantages of Babinet's compensator over the $\lambda/4$ plate.

(c) Explain how continuous variation in retardance of a wave plate can be achieved.

(d) Calculate the thickness of a half-wave plate for a light of wavelength 550 nm. Given the refractive indices $n_o = 1.5442$ and $n_e = 1.5533$. 2+(2+1)+2+3

7. (a) Imagine two crossed linear polarizers with transmission axes vertical and horizontal. Now insert a third linear polarizer between them with transmission axis at 45° to the vertical. Determine the intensity of the emerging wave in terms of intensity of the incident wave.

(b) The rotation in the plane of polarization at wavelength 589.3 nm in a certain substance is 10°/cm. Calculate the difference between the refractive indices for right and left circular polarized light in the medium. Derive the equation that you use.

(c) Define Optic axis, Principal section and Principal plane for a double refracting crystal. 3+4+3

Syllabus : 2018-2019

(Quantum Mechanics and Applications)

Full Marks : 50

Answer ***question no.*** **1** and ***any four*** questions from the rest.

2×5

1. Answer ***any five*** questions :

(a) A particle moves in a linear potential $V(x) = (-kx)$. Show that $\langle x \rangle$ follows the same time evolution as given by Newton's law.

(b) Prove that total wave function for Bosons is symmetric.

(c) In Stern-Gerlach experiment, a beam of atoms is passed through an inhomogeneous magnetic field. What will happen if ions are used instead of atoms.

(d) If $V(x)$ is an even function, then wave function can always be taken to be either even or odd. Justify.

(e) Show that the quantity $y = \sqrt{\alpha}x$, where $\alpha = \frac{m\omega}{\hbar}$ as used in a harmonic oscillator is dimensionless.

(f) Find the magnetic moment of an atom in the state $2D_{3/2}$.

(g) Find the quantum number corresponding to an oscillator of mass 2 gm, angular velocity 1 rad/Sec, amplitude 1 cm & using correspondence principle, comment on its nature.

2. (a) A stream of particles of mass m energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x > 0$. If the energy of the particles $E > E_0$, show that the sum of fluxes of the transmitted and reflected particles is equal to flux of incident particles.

(b) The wave function of linear harmonic oscillator in the potential $V(x) = \frac{1}{2}mw^2x^2$ at a given instant is $\psi(x) = \left(\frac{2}{\pi\beta^2}\right)^{\frac{1}{4}} e^{-x^2/\beta^2}$ For what value of β will the function $\hat{a}\,\psi(x)$ be zero?

[Given : $\hat{a} = \frac{1}{\sqrt{2}}\left[\hat{x} + i\hat{p}_x\right]$

(c) Prove that energy eigenfunction of a free particle is doubly degenerate. 5+3+2

3. (a) The vector space for a particle of spin half has basis $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ in which s_z is diagonal having eigenvalues ½ and –½ respectively. Using this basis find the normalized eigenvector of S_y with eigenvalues (–½).

(b) Consider the wave function

$\psi(x,0) = Ae^{-\lambda|x|}e^{-iwt}$

(i) Normalize ψ

(ii) Determine the expectation value of x.

(iii) Find the standard deviation of x.

(c) Discuss the changes that the confined particle observes if the infinite square will is made finite. 3+(1+2+2)+2

4. (a) Write down the schrödinger equation for the electron of Tritium (H_3) atom, assuming the nucleus to be stationary. Obtain the radial equation by separation of variables with special emphasis on effective potential.

(b) Find $\langle S_z \rangle$ in the state $X = A\begin{pmatrix}1-2i\\2\end{pmatrix}$.

(c) Show that $L_{\pm}f_l^m = \hbar\sqrt{l(l+1) - m(m+1)}\,f_l^{m\pm1}$ where the symbols have their usual meaning. 4+3+3

5. (a) What is Lande-g factor? Obtain an expression in terms of l, s, j.

(b) What is the lande-g-factor for an atom with a single optical electron in $d_{3/2}$ level?

(c) Find out the different factors in the J-J coupling. Scheme for a two electron atom given by $l_1 = 1$ and $l_2 = 2$. (1+3)+2+4

6. (a) Write down the fine structure formula mentioning each correction term with respect to Bohr energy of hydrogen atom.

(b) Using 1st order perturbation theory, obtain the relativistic correction term to the kinetic energy.

(c) Using uncertainty principle estimate the ground state energy of hydrogen atom. 3+4+3

7. (a) An atomic state is denoted by $4D_{5/2}$. Give the values of L, S and J. What should be the minimum number of electrons involved for this state? Give a possible electronic configuration.

(b) In K, the longest wavelength lines in $(n, 1) \rightarrow (4, 0)$ transitions have wavelength 7699, 7665, 4047 and 4044 Å respectively. Find the splitting between the levels in the same n and l but different j.

(c) A source of light is placed between the poles of an electromagnet. What will you observe if the light is examined by a spectroscope in directions parallel and perpendicular to the magnetic field. Assume the magnetic field to be not too strong. Give an explanation of the phenomenon. 4+3+3

2022

PHYSICS — HONOURS

Paper : CC-12

(Syllabus : 2019-2020)

[Statistical Physics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) A linear harmonic oscillator has energy in the interval E to $E + \Delta E$. By means of a suitable diagram, demarcate the region of phase space where the oscillator has all its possible microstates.

(b) State the postulate of equal a priori probability.

(c) What is λ-transition in liquid Helium?

(d) Sketch the spectral distribution graphs (u_λ vs. λ) of black-body radiation for two different temperatures T_1 and T_2 ($T_2 > T_1$). Do they intersect at any value of λ?

(e) Show that for a canonical system, mean energy

$$\langle E \rangle = -\frac{\partial}{\partial \beta}(\ln Z),$$

where Z is the canonical partition function.

(f) Show that the single particle occupation probability for a Fermion at T K with an energy interval $\pm kT$ around Fermi energy has an approximate range of 0.46.

(g) The concentration of electron in Cu is $8.5 \times 10^{28}\ m^{-3}$. Calculate whether the electron gas in Cu is degenerate or non-degenerate at $T = 500$ K.

(h) In how many ways can five Bosons be arranged in three quantum states?

2. (a) From the consideration of statistical mechanics under what condition will system 'A' (with volume 'V_A', energy 'E_A', number of particles 'N_A' and number of microstates Ω_A) only capable of exchanging energy with another system 'B' (with volume 'V_B', energy 'E_B', number of particles 'N_B' and number of microstates Ω_B) achieve equilibrium when the composite system consisting of the two systems is an isolated system?

(b) If in addition to energy the two systems can perform work on each other, what would be the condition of equilibrium?

(c) Assuming the entropy S is a function of the number of microstates Ω, show that the additive property of S and the multiplicative property of Ω necessarily require $S \propto \ln \Omega$. 4+2+4

3. Consider a classical ideal gas consisting of N identical and indistinguishable molecules of mass m enclosed in a container of volume V.

   (a) Calculate single particle partition function (ξ) and hence write down the partition function (Z) for the system by taking into account the fact that gas molecules are indistinguishable.

   (b) Calculate the mean energy of the system.

   (c) Calculate Helmholtz free energy of the system and hence derive Sackur Tetrode equation.

(3+1)+2+(2+2)

4. (a) A particle in 1D has energy

$$E = \frac{p^2}{2m} + \lambda q^4,$$

where q and p denote the generalized coordinate and momentum, respectively. Show that heat capacity of a gas comprising of N such particles is

$$C_V = \frac{3}{4} Nk.$$

   (b) The grand canonical partition function for an ideal gas is given by

$$Z_G (T, V, \mu) = \exp\left[e^{\mu/kT} \frac{V}{\lambda^3} \right],$$

where $$\lambda = \frac{h}{\sqrt{2\pi mkT}}$$

is the thermal de Broglie wavelength. Calculate average number of particles and hence calculate the equation of state of the system.

   (c) Why microcanonical, canonical and grand canonical ensembles give almost same results for a system with large number of particles?

4+3+3

5. (a) From Bose-Einstein distribution function, obtain Planck's law of black-body radiation. Hence derive Wein distribution law and Rayleigh-Jeans law.

   (b) What do you mean by ultraviolet catastrophe?

   (c) Assuming Sun radiates maximum energy at λ = 6000Å, calculate the approximate wavelength at which human body radiates maximum energy. Temperature of Sun is 6000°C.

(4+2)+2+2

(symbols have their usual significance)

6. (a) Consider two indistinguishable particles (1 and 2) which may exist in two different states ('a' and 'b'). The wave function for the entire system will be one of the following two possible forms :

$$\Psi_1 = \frac{1}{\sqrt{2}} [\psi_a (1) \psi_b (2) + \psi_a (2) \psi_b (1)] \text{ and } \Psi_2 = \frac{1}{\sqrt{2}} [\psi_a (1) \psi_b (2) - \psi_a (2) \psi_b (1)].$$

Identify with reason the wave function that will represent the system when (i) both are Bosons, (ii) both are Fermions.

(b) Consider a system of two identical fermions which may occupy any of the three energy states 0, ε and 2ε. The system is in thermal equilibrium at absolute temperature T. Determine the partition function and the average energy of the system.

(c) Which statistics (Bose-Einstein or Fermi-Dirac) will apply to—

(i) alpha particles

(ii) Helium-3 atoms

(iii) Deuterium nuclei

(iv) Neutrons? 4+4+2

7. (a) Calculate the average energy of a strongly degenerate Fermi gas at $T = 0\,\text{K}$.

(b) The specific heat of silver at low temperatures can be represented by $C_V = \gamma T + \alpha T^3$ where γ and α are constants. Explain the origin of the term linear in T.

(c) In terms of number density (n) and absolute temperature (T), find the condition to be satisfied by an ideal gas for the onset of quantum effects. 5+3+2

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(Syllabus : 2018-2019)

[Solid State Physics]

Full Marks : 50

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer *any five* questions : 2×5

   (a) Calculate the density of atoms in (111) planes of fcc aluminium whose lattice parameter is 4.05Å.

   (b) Find out the packing fraction of a simple cubic structure.

   (c) Is Meissner effect consistent with the disappearance of resistivity in a superconductor? Explain.

   (d) With the help of M-H plot distinguish between type-I and type-II superconductors.

   (e) Will the Hall coefficient change sign if one reverses the direction of the applied magnetic field?— Explain.

   (f) Sketch the spontaneous magnetization of a ferromagnet as a function of temperature. Indicate the universal feature associated with the graph.

   (g) The Fermi level for potassium is 2.1 eV at $T = 0K$. What is the classical and quantum average energy of the electrons at $T = 0K$?

2. (a) Write down the Laue equations and arrive at the Bragg's Law therefrom.

   (b) What is a reciprocal lattice and how do you construct it?

   (c) A beam of thermal neutrons emitted from the opening of the reactor is diffracted by the (111) planes of nickel crystal at an angle of $28°30'$. Calculate the effective temperature of the neutrons. Nickel has fcc structure and its lattice parameter 3.52Å. 3+(2+1)+4

3. (a) Starting from the equation

   $$\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a = \cos Ka$$

   where $P = \frac{mV_0ab}{\hbar^2}, \alpha^2 = \frac{2mE}{\hbar^2}$ and symbols have their usual meaning. Find the lowest energy band using Kronig-Penney model for $P << 1$.

   (b) Schematically represent the variation of velocity, effective man, and acceleration of an electron moving in a periodic potential.

   (c) The Hall coefficient of a specimen is -7.35×10^{-5} m^3c^{-1}. What is the nature of the semiconductor? If the conductivity is 200 $m^{-1}\Omega^{-1}$, calculate the density and mobility of the charge carrier. 4+3+(1+2)

4. (a) The phonon dispersion relation for a vibrating diatomic chain in which alternate atoms are M_1 and M_2 is given by

$$\omega^2 = K_1\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm K_1\left[\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4\sin^2 Ka}{M_1 M_2}\right]^{\frac{1}{2}}$$

(K_1 = force constant; $K = \frac{2\pi}{\lambda}$)

Identify and obtain the minimum and maximum angular frequencies of the acoustical and optical branch.

(b) How does Debye model differ from the Einstein model of lattice heat capacity?

(c) Visible light of wavelength 5000 Å undergoes scattering from a crystal of refractive index 1.5. Calculate the maximum frequency of the phonon generated. (2+2)+2+4

5. (a) A paramagnetic atom having permanent moment $\vec{\mu}$ with a given resultant quantum number $\vec{J}$ is placed in a uniform magnetic field $\vec{B}$. Obtain an expression of the magnetisation as a function of $\vec{B}$ and temperature T. Hence obtain Curie's Law in the appropriate limit.

(b) Explain the hysteresis phenomena on the basis of domain theory.

(c) Atomic weight and density of iron are 55.847 and 7.87×10^3 kg m^{-3} respectively. If iron has a magnetic moment of 2.2 Bohr magneton, determine its spontaneous magnetization.

(4+1)+3+2

6. (a) What do you mean by polarization of a solid?

(b) Clearly explain the basic assumptions and derive the Clausius-Mossotti relation for a dielectric. Explain how it is modified when more than one dielectric is present.

(c) The polarizability of NH_3 molecule in the gaseous state from the measurement of dielectric constant is found to be 2.42×10^{-39} Fm^2 at 309K and 1.74×10^{-39} Fm^2 at 448K. Compute the polarizability due to permanent dipole moment of NH_3 at 448K. 1+(4+2)+3

7. (a) What are Cooper pairs? How are they formed inside the superconducting material? What can you say about its spin state? How can you destroy the Cooper pairs?

(b) What is the importance of penetration depth in superconductivity? The penetration depths for lead are 396Å and 1730 Å at 3K and 7.1K respectively. Calculate the critical temperature for lead.

(1+3+1+1)+(1+3)

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2022

PHYSICS — HONOURS

Paper : DSE-A-1(a) and DSE-A-1(b)

The figures in the margin indicate full marks.

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Paper : DSE-A-1(a)

(Advanced Mathematical Methods)

Full Marks : 65

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest (***Q.*** **3** to ***Q.*** 8).

1. Answer ***any five*** questions : 2×5

(a) Prove that any set of vectors that include the null vector is linearly dependent.

(b) Is it possible to write $v = (1, -2)$ in $\mathbb{R}^2$ as a linear combination of the vectors $u_1 = (1, -3)$ and $u_2 = (2, -4)$?

(c) What is a 'functional'? Show that for any vector $|a\rangle$ of a complex vector space $V(C)$, $f(|v\rangle) = \langle a|v\rangle$ defines a linear functional for $|v\rangle \in V(c)$.

(d) How do the contravariant and the covariant components of a vector transform under a general co-ordinate transformation?

(e) The moment of inertia tensor of a square lamina, having mass 'M' and sides of length 'L' is given by :

$$\begin{pmatrix} ML^2/3 & -ML^2/4 & 0 \\ -ML^2/4 & ML^2/3 & 0 \\ 0 & 0 & 2ML^2/3 \end{pmatrix}$$, where its two adjacent sides are taken

as the X– and Y– axis. Find its moment of inertia about a diagonal.

(f) Show that in an Abelian group, every element is a (conjugacy) class by itself.

(g) All the elements of a group G_1 is mapped to a single element of another group G_2. Can it be a homomorphism?

2. Answer ***any three*** questions :

(a) Prove that : $|\langle a|b\rangle| \leq \|a\| \cdot \|b\|$ for any two complex vectors $|a\rangle, |b\rangle$, where $\|a\|$ and $\|b\|$ stands for their norms. 5

(b) (i) Express the scalar triple product of three vectors, using the Levi-Civita symbol and hence prove that : $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\vec{B}\cdot\left(\vec{C}\times\vec{A}\right)$.

(ii) From the equation $\partial_\mu F^{\mu\nu}=J^\nu$, where $F^{\mu\nu}$ is the electromagnetic field strength tensor, and J^ν is the 4-current density, obtain the equation of continuity in the three-vector form. 2+3

(c) Prove that, if A^μ are the contravariant components of a 4-vector, then $A_\mu=g_{\mu\nu}A^\nu$ transform in the covariant fashion, where $g_{\mu\nu}$ is the 4-dimensional metric tensor. How do you define $g^{\mu\nu}$? 4+1

(d) Let (G, o) be a group and $H \subseteq G$. Prove that, if $h_1, h_2 \in H \Rightarrow (h_1 \bullet h_2^{-1}) \in H$, then H is a sub group of G. 5

(e) Consider a two-object permutation group. Construct its group multiplication table and show that this is an abelian group. Construct a valid matrix representation of the elements of the group. 2+1+2

3. (a) Check whether the set of matrices

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}1&0\\0&-1\end{pmatrix},\begin{pmatrix}0&-1\\1&0\end{pmatrix}\text{ and }\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

is linearly independent.

(b) A linear operator $\mathbb{R}^3 \to \mathbb{R}^3$ takes $\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$ to $(x+2y+z)\hat{i}+(x-y+2z)\hat{j}+(3x-y+z)\hat{k}$.

Find the matrix representation of the operator.

(c) Let V be the vector space over all 2×2 matrices over the real field R. Show that W is not a subspace of V, where

(i) W consists of all matrices with zero determinant.

(ii) W consists of all matrices for which $A^2=A$. 3+3+(2+2)

4. (a) Define an 'inner product space'.

(b) Show that a set of orthogonal non-null vectors (i.e., each pair of distinct vectors in the set is orthogonal) is always linearly independent.

(c) Inner product is defined over a vector space of functions as : $\langle f(x)\,|\,g(x)\rangle=\int_{-\infty}^{\infty}e^{-x^2}f(x)\,g(x)dx$.

Use Gram-Schmidt procedure to orthogonalize the set of functions : $\{1, x, x^2\}$ with respect to the inner product defined above. (The resultant functions need not be normalized.) 3+2+5

5. (a) Derive an expression for the moment of inertia tensor of a general body.

(b) Three particles, each of mass 'm' are situated at the points : (1,1,1), (–1,1,2) and (0,–1,–2), Construct the moment of inertia tensor for the system. 5+5

6. (a) What is a 'four vector'? Write the contravariant components (p^μ) and the covariant components (p_μ) of the momentum 4-vector and evaluate the 4-scalar p^2. (take the four-dimensional metric tensor $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$).

(b) How are the components of the electric and the magnetic field related to those of the electromagnetic field strength tensor $F^{\mu\nu}$?

Show that the relation : $\partial_\mu F^{\mu\nu} = J^\nu$ produces the inhomogeneous Maxwell's equations, while expressed in 3-vector notation. (1+1+1+1)+(2+4)

7. (a) Show that the same element cannot occur twice in a column of a group multiplication table.

(b) The multiplication table of a group is given below :

	E	R	R^2	σ_A	σ_B	σ_C
E	E	R	R^2	σ_A	σ_B	σ_C
R	R	R^2	E	σ_C	σ_A	σ_B
R^2	R^2	E	R	σ_B	σ_C	σ_A
σ_A	σ_A	σ_B	σ_C	E	R	R^2
σ_B	σ_B	σ_C	σ_A	R^2	E	R
σ_C	σ_C	σ_A	σ_B	R	R^2	E

(i) Find the inverses of the elements R and σ_A.

(ii) Find the conjugacy classes of R and σ_A. 2+(2+6)

8. (a) Consider a 2-dimensional anticlockwise rotation by an angle θ that takes the orthogonal (x, y) axes to (x', y').

(i) How many parameters are needed to parametrise this group? Is it an abelian group?

(ii) Find the matrix representation X of the generator of this group.

(iii) Show that $e^{i\theta x} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

(b) Consider the algebra of a Lie group

$[X_a, X_b] = i f_{abc} X_c$

where f_{abc} s are all real. Show that

(i) $f_{abc} = -f_{bac}$ for all values of a and b.

(ii) Show that $-X_a^*$ matrices also satisfy the same Lie algebra. (2+2+2)+(2+2)

Paper : DSE-A-1(b)

(Laser and Fibre Optics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1** and **2** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Calculate the numerical aperture and acceptance angle of an optical fibre for core and cladding refractive indices 1.51 and 1.48 respectively.

(b) What do you mean by monochromaticity of an optical beam?

(c) Find the intensity of a laser beam of 15 mW power and having a diameter of 1.3 mm. Assume the intensity to be uniform across the beam.

(d) Why a three-level laser normally provides pulsed output?

(e) Distinguish between step index fibre and graded index fibre structure.

(f) What is Pockels effect?

(g) What is meant by self-focussing?

2. Answer ***any three*** questions :

(a) A step index fibre is made with a core of index 1.52, a diameter of 30 μm and a fractional difference of index 0.0007. It is operated at a wavelength 1.3 μm. Find

(i) The V-number of fibre and

(ii) The approximate number of modes the fibre can carry. 3+2

(b) Explain the principle of holography with diagram. 5

(c) A two-level laser has n_1 and n_2 number of atoms/unit volume in level 1 and level 2 respectively. Then show that

$$\Delta n = \frac{n_o}{1+2P\tau}, \text{ where } n_o = n_1 + n_2, \quad \Delta n = n_1 - n_2$$

τ is the overall lifetime of upper level and P is pumping rate. 5

(d) Show that (i) confocal resonator and (ii) planer resonator can act as stable optical resonator. Write down the necessary equation. Hence, draw the stability diagram and indicate the positions of the above mentioned resonators in the diagram. 1+1+1+1+1

(e) (i) With proper schematic diagram discuss how the Fabry Perot etalon can help in mode selection in an optical resonator.

(ii) The cavity of He-Ne laser emitting at 632.8 nm consists of two mirrors separating a distance of 35 nm. If the oscillation in the laser cavity occurs at frequency within the gain bandwidth of 1.3 GHz, then find out the number of longitudinal modes allowed in the cavity. 3+2

3. (a) Find an expression for the intensity of Second Harmonic Generation at the exit surface of a material.

(b) From the expression for the intensity obtain the criterion for phase matching.

(c) Why is it called refractive index criterion? 6+2+2

4. (a) Deduce the relation, with a suitable diagram, between the Einstein's A and B coefficients.

(b) At thermal equilibrium, obtain the ratio of the number of spontaneous to stimulated emissions.

(c) A signal of 100 mW is injected into a fibre. The outcoming signal from the other end is 40 mw. Find the loss in decibel (dB). 6+2+2

5. (a) Let a step index single mode fibre is characterised by

$$n(r) = n_1 \text{ for } 0 < r \le a \text{ (core)}$$
$$= n_2 \text{ for } r > a \text{ (cladding)}$$

where $n(r)$ is refractive index, k_0 = free space wave number, β = the wave propagation constant and a = radius of the core. Show that the guided mode is possible when $n_2^2 < \frac{\beta^2}{k_0^2} < n_1^2$.

(b) In the case of multimode graded index fibre, using power law profile, show that a parabolic index fibre can accommodate nearly 25 modes.

(c) A step index fibre with $n_{core} = 1.485$ and $n_{cladding} = 1.455$ has a core radius, $a = 5.92\times10^{-6}$ m. Calculate the operational wavelength (λ_0) for which waveguide parameter, $V = 10$. 6+2+2

6. (a) Define quality factor in a resonator cavity.

(b) If W_0 represents the energy stored in a mode at $t = 0$, then find out the expression for the energy stored in that mode at time t in terms of quality factor.

(c) Hence find out the passive cavity lifetime.

(d) If the resonator has two end mirrors M_1 and M_2 placed at a distance d from each other with reflectivity R_1 and R_2, write down the expression for energy stored in the mode which corresponds to a pair of reflection. (α_C is the net power absorption coefficient and n_0 is the refractive index of the active medium).

(e) Find out the quality factor and passive cavity lifetime in terms of R_1, R_2, n_0, d and α_C. 2+2+1+1+2+2

7. (a) Write down the allowed frequencies of the optical modes in a rectangular cavity of dimension $2a \times 2b \times d$. Hence obtain the expression for the frequencies in case of an open resonator with proper approximation.

(b) Show that the difference of frequency between two longitudinal mode is $C/2d$ for $a = b$.

(c) Find out the frequency difference between two consecutive transverse mode for a single longitudinal mode. 2+3+2+3

8. (a) Write down the rate equations for a 3-level laser explaining the symbols.

(b) Obtain the condition for population inversion from the above equations.

(c) For a Ruby laser, number of atoms per cm^3 is 1.6×10^{19}, spontaneous lifetime is 3×10^{-10}s and average pump frequency is 6.25×10^{14} Hz. Calculate the threshold pump power. 3+5+2

2022

PHYSICS — HONOURS

Paper : DSE-B-1(a) and DSE B-1(b)

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Paper : DSE-B-1(a)

(Astronomy and Astrophysics)

Full Marks : 65

Answer ***question nos.*** **1** and **2** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) The luminosity of a star is 40 times that of the Sun and its temperature is twice as much. Determine the radius of the star in terms of solar radius.

(b) If the apparent magnitudes of the components of a binary star are 1 and 2, what will be the total apparent magnitude of the system? (*hint* : it's not 3, unlike the flux we cannot simply add magnitudes.)

(c) Show that the specific intensity along the ray in free space is constant.

(d) For the same diameter, compare the resolving power of an optical telescope operating at $\lambda = 457$ nm (nanometer) and a radio telescope operating at $\lambda = 1$ cm.

(e) What is meant by Local Thermodynamic Equilibrium?

(f) Argue that the p-p chain reaction is somewhat capable to explain the fact that the sun is still shining beyond the Kelvin-Helmholtz time scale.

(g) Consider a carbon white dwarf of mass 1.4 M_{sun}, the Chandrasekhar mass. How many ^{12}C nuclei are there?

2. Answer ***any three*** questions :

(a) At one stage during its birth, the protosun had a luminosity of 1000 $L_{\odot}$ and a surface temperature of about 1000 K. At this time, what was its radius? Express your answer in three ways : as a multiple of the Sun's present-day radius, in kilometers, and in astronomical units (AU). 5

(b) Briefly discuss the different stages of stellar evolution, starting from the main sequence to creation of white dwarfs, neutron stars and black holes along with their observational evidences. 5

(c) Explain briefly the possibility of nuclear fusion inside the core of stars irrespective of the fact that two nuclei of same charge repel each other due to Coulomb interaction. 5

(d) Consider the value of solar luminosity to be 3.9×10^{26} W, and four protons are required to produce a helium nucleus releasing about 26 MeV of energy. Estimate the number of protons used per second to account for the observed luminosity. Also estimate the flux of neutrinos on earth due to nuclear reactions inside the sun. (Neglect pp2 and pp3 branches, and use 1 AU $= 1.5\times10^{11}$m.). 3+2

(e) Use the Friedmann equation and the fluid equation to derive the acceleration equation for an expanding universe. 5

3. (a) The mass of star Sirius is thrice that of the Sun. Find the ratio of their luminosities and the difference in their absolute magnitudes. Taking the absolute magnitude of the Sun as 5, find the absolute magnitude of Sirius. (Use the scaling relation $L \sim M^{3.5}$, Sirius is 47 times more luminous than Sun; $M_{\text{Sirius}} = 0.83$)

(b) Calculate the diffraction limit of resolution of Mount Palomar telescope of 200 inch diameter for $\lambda = 457$ nm. Compare its light gathering power with a telescope of 200 mm diameter.

(c) Show that pressure due to isotropic radiation field is given as $p_\nu = \dfrac{4\pi I_\nu}{3c}$. 3+3+4

(Symbols have their usual meanings)

4. (a) From the first principles of statistical mechanics, obtain the scaling relationship between pressure and density of a degenerate fermi gas in non-relativistic limit.

(b) What are the limitations of Chandrasekhar's theory?

(c) What are brown dwarfs? 5+2+3

5. (a) Suppose a star has a density profile $\rho(r) = \dfrac{A}{r(r_s + r)^2}$. Find out the mass contained within a radius r. (A and r_s are constants.)

(b) Find out the speed of rotation $v(r)$ and plot it as a function of r.

(c) Show that for $r << r_s, v \sim \sqrt{r}$. 3+(2+2)+3

6. (a) Derive the hydrostatic equilibrium equation of stellar structure for spherically symmetric objects under the assumption that accelerations are negligible.

(b) Consider a sun-like star with mass $M = 1.99\times10^{30}$kg and radius $R = 6.96\times10^{8}$ m.

(i) From the hydrostatic equilibrium equation for star, calculate the pressure at radius $r = R_\odot/2$.

(ii) Estimate its temperature at $r = R_\odot/2$ (take the mean molecular weight of the star $\mu = 0.61$).

(iii) Calculate the radiation pressure in the star at $r = R_\odot/2$.

(take the radiation constant $a = 7.57\times10^{-16}$ J/m^{3}K^4) 4+(3+2+1)

7. (a) The two Friedmann equations for a simple flat universe are given as

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)$$

where R is the scale factor, ρ is the mean mass density and P is the pressure of the Universe.

(i) From the above equations derive the *fluid equation* as

$$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{P}{c^2}\right) = 0.$$

(ii) Now consider only radiation with pressure $P = \frac{1}{3}\rho c^2$ and show that $\rho \propto R^{-4}$. What is the physical explanation for density falling as the fourth power of scale factor?

(iii) Using Friedmann equation, show that the Hubble parameter for this universe will be

$$H(t) = \frac{1}{2t}.$$

(b) What is *Einstein-de Sitter* Universe? Considering a small expanding spherical region in this universe, show that mean density of the universe (known as the *critical density* ρ_c) is $\rho_c = \frac{3H^2}{8\pi G}$.

(2+3+2)+(1+2)

8. Consider a spherical region of the universe having total mass $M = \frac{4\pi r^3}{3}\rho$, with density ρ, centred on a given point of radius r at a time t. Take a test galaxy of mass m at the edge of that massive sphere. It will be affected by a central force due to gravity and the cosmological force as :

$$m\ddot{r} = -\frac{4\pi G}{3}\frac{r^3\rho m}{r^2} + \frac{1}{3}m\Lambda r$$

Note that, both r and ρ are changing with time and can be expressed in terms of scale factor R as $r = (R/R_0)r_0$ and $\rho = (R_0/R)^3\rho_0$.

(a) Show that,

$$d\left(\dot{R}^2\right) = -\frac{2a}{R^2}dR + \frac{2}{3}\Lambda R dR$$

where $a = 4\pi G R_0^3 \rho_0 / 3$.

(b) From there arrive at the basic differential equation governing $R(t)$:

$$\frac{\dot{R}^2}{H_0^2 R_0^2} = \Omega_0 \frac{R_0}{R} + \Omega_\Lambda \left(\frac{R}{R_0}\right)^2 + 1 - \Omega_0 - \Omega_\Lambda$$

where $\Omega_0 = \rho_0/\rho_c$ and $\Omega_\Lambda = \Lambda/(3H_0^2)$. The critical density $\rho_c = 3H_0^2/8\pi G$.

(c) Now take a simple Universe with $\Omega_\Lambda = 0$ and $\Omega_0 = 1$. By solving the equation for $R(t)$, show that the age of this Universe is $t_0 = \frac{2}{3}\frac{1}{H_0}$. 3+4+3

Paper : DSE-B-1(b)

(Nuclear and Particle Physics)

Full Marks : 65

Answer ***question nos.*** **1**, and **2** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Which of the nuclei between $^{54}_{27}\text{Co}$ and $^{55}_{27}\text{Co}$ do you expect to have higher neutron capture cross-section and why?

(b) Justify that the time scale of direct nuclear reactions is of the order of 10^{-22} s.

(c) Compare the stopping power of a proton and an alpha-particle of same energy in a particular medium.

(d) Why is the scintillation detector more efficient than the GM counter for the detection of γ-ray?

(e) Give an experimental evidence of short range nature of nuclear force.

(f) Neutron is electrically neutral, but has a magnetic moment $-1.91\ \mu_N$. Explain the reason.

(g) Find the distance of closest approach of 1 MeV proton incident on gold nucleus ($Z = 79$) with zero impact parameter.

2. Answer ***any three*** questions :

(a) What are mirror nuclei? Give an example. Find out the mass difference between two mirror nuclei in terms of their mass number by using the semi-empirical mass formula. 2+3

(b) What do you mean by stripping nuclear reaction? Give an example. Define the Q value of nuclear reaction in terms of the nuclear binding energy and thereby calculate the Q value of the $^{27}Al(n,p)^{27}Mg$ reaction. Given, the binding energies of ^{27}Al and ^{27}Mg are 224.95 MeV and 223.76 MeV respectively. 2+3

(c) Using a schematic circuit diagram, explain the working principle of a semiconductor detector for detecting nuclear radiation. What are the main advantages of semiconductor detectors amongst all types of radiation detectors? 3+2

(d) Why is it necessary to increase the length of the successive tubes in a linear accelerator (LINAC)? Show that the length of the nth tube of a LINAC is proportional to $\sqrt{n}$. 2+3

(e) Mention the distinct characteristics of the weak interaction. Are the parity and strangeness conserved in weak interaction? Give an example of a phenomenon involving the weak interaction. 3+1+1

3. (a) Calculate the nuclear surface area and estimate the Coulomb repulsion energy of ^{17}O nucleus. Determine its spin and parity in the ground state.

(b) $^{64}_{28}Ni$ and $^{64}_{29}Cu$ have atomic masses 63.9280 u and 63.9298 u respectively. Which of these nuclei exhibits β disintegration and of what type? Justify your answer. Given, the mass of electron is 0.0005 u. (2+2+2)+4

4. (a) Consider the following reaction where X is the target nucleus at rest, a is the projectile, Y is the residual nucleus and b is the outgoing particle.

$$a + X \rightarrow Y + b$$

Draw the appropriate vector diagram of the above reaction in the centre of mass frame.

(b) Suppose you would like to produce $^{92}_{40}Zr$ as compound nucleus using $^{76}_{32}Ge$ as target and $^{16}_{8}O$ as projectile. What should be the minimum beam energy (kinetic energy) of $^{16}_{8}O$?

(c) A nucleus (X) undergoes α decay by emitting two groups of α-particles of different energies accompanying by γ-radiation. Represent this process in the energy level diagram. 3+4+3

5. (a) What is Cherenkov radiation? Show that the necessary condition for Cherenkov radiation is $v > c/n$, where v is the velocity of the particle, c is the velocity of light in free space and n is the refractive index of the medium.

(b) What is the typical energy of the electron emitted in the β disintegration? How such an electron loses energy while passing through a medium?

(c) The absorption cross-section of 1 MeV gamma rays in lead (mass number 208, density 11.25 g/cc) is 20 barns/atom. Check whether a lead block of thickness 7 cm will act as shield for 1 MeV gamma rays. (1+3)+(1+2)+3

6. (a) Derive an expression for the maximum energy of acceleration attainable in a fixed frequency cyclotron. Is it possible to use the same cyclotron for accelerating deuterons and α-particles? Explain. In which case do you expect higher energy of acceleration?

(b) Why is it not possible to accelerate the ions to their relativistic energies with fixed frequency cyclotron? How is this limitation circumvented with a synchro-cyclotron? (3+2+1)+(2+2)

7. (a) Describe the working principle of a GM counter for the detection of nuclear radiation. Why is it not possible to detect the type and measure the energy of the incident radiation by using GM counter?

(b) Why a meson with charge +1 and strangeness –1 is not possible?

(c) What is strange about strange particles? (4+2)+2+2

8. (a) Using the conservation of lepton number, find out the missing particles in the following muon decay :

$$\mu^- \to e^- + \ldots + \ldots$$

(b) Explain the eightfold way in reference to the meson octet.

(c) An isospin singlet baryon has strangeness of –3. Determine the hypercharge and electric charge of the baryon. Do you identify the baryon?

(d) Are the following reactions allowed or forbidden? Justify your answer.

(i) $\Xi^- \to n + \pi^-$

(ii) $\pi^- + n \to \Sigma^- + K^0$. 2+3+2+3

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