2022

PHYSICS — HONOURS

(Syllabus : 2019-2020 & 2018-2019)

Paper : CC-1

[Mathematical Physics - I]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Evaluate $\underset{x\to4}{\mathrm{Lt}}\dfrac{\sqrt{x}-2}{x-4}$, if it exists.

(b) Plot schematically xe^{-x} vs. x for $0 \le x < \infty$.

(c) Find whether vectors $2\hat{i}+5\hat{j}+3\hat{k}, \hat{i}+\hat{j}+\hat{k}$ and $4\hat{i}-2\hat{j}$ are linearly independent.

(d) Find the Taylor series expansion of $\ln x$ about $x = 2$.

(e) Determine Wronskian of the two solutions to the following differential equation $x^4y''-2x^3y'-x^8y=0$.

(f) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$.

(g) Prove that the eigenvalue of a skew Hermitian matrix is purely imaginary.

2. (a) Plot the function $f(x) = x^2$ and its first derivative.

(b) Find the constant term in the expansion of $\left(x^2+\dfrac{1}{x^2}\right)^6$.

(c) Check whether $df = (3x^2 - 3ay)dx + (3y^2 - 3ax)dy$ is an exact differential.

(d) Find the series expansion of $\dfrac{1}{1-x}$. Mention its interval of convergence. 2+2+2+(2+2)

Or,

(For Syllabus : 2018-2019)

(d) Consider

$$f(x) = \begin{cases} x^m \sin\left(\dfrac{1}{x^n}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Find out the condition on m and n so that $f(x)$ is differentiable at $x = 0$. 4

3. (a) Solve the equation :

$$e^x \sin y\, dx + (e^x + 1)\cos y\, dy = 0$$

(b) Check whether the function $\sin x$, e^x and e^{-x} are linearly independent or not.

(c) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box that requires minimum surface area for its construction. 3+3+4

4. (a) Find the directional derivative of $\phi = x^2y + xz$ at (1,2,–1) in the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$.

(b) If $\vec{\nabla} \times \vec{F} = 4x\hat{i} - 2x\hat{j} + cz\hat{k}$ then find c.

(c) Consider two vector fields $\vec{F_1} = 2x\,\hat{i} - 2yz\,\hat{j} - y^2\hat{k}$ and $\vec{F_2} = y\hat{i} - x\,\hat{j}$. Which of the above is a conservative field? For the non-conservative field, calculate the work done if it acts on an object moving from (–1, –1) to (1, 1) along the straight line joining the two points. 3+2+(2+3)

5. (a) A fluid motion is given by $\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Show that the motion is irrotational and hence find the velocity potential.

(b) Use Green's theorem to evaluate

$$\oint_c [(xy + y^2)dx + x^2dy]$$

where c is the triangle with vertices (0, 0), (1, 0) and (1, 2).

(c) Evaluate $\iint \vec{A}.d\vec{s}$ where $\vec{A} = x\cos^2 y\,\hat{i} + xz\,\hat{j} + z\sin^2 y\,\hat{k}$ over the surface of a sphere with centre at the origin and of radius 3 unit. 3+4+3

6. (a) The matrices A and B satisfy $(AB)^T + B^{-1}A = 0$. Prove that if B is orthogonal, then A is anti- symmetric.

(b) Find out the eigenvalues and normalized eigenvectors of the matrix $M = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} (a \neq 0)$. Find out M^n where 'n' is a positive integer.

(c) Explain whether the inverse of the following matrix exists $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$. 2+(1+2+2)+3

7. (a) If $A^2 = A$, then show that $e^{\theta A} = \mathbb{I} + (e^{\theta} - 1)A$.

(b) Given $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$; what can you comment on the nature of eigenvalues of A without solving the characteristic equation.

(c) Show that two similar matrices have the same characteristic polynomial.

(d) Show that for a orthogonal matrix, each column is orthogonal to other ones. 2+2+3+3

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2022

PHYSICS — HONOURS

[For Syllabus : 2019-2020 & 2018-2019]

Paper : CC-2

(Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) Show that Newton's laws of motion remain invariant under Galilean transformation.

(b) A particle moves under the influence of a force $\vec{F}$ and has an instantaneous velocity $\vec{V}$. Show that $\frac{dT}{dt} = \vec{F}.\vec{V}$, where T is the kinetic energy of the particle.

(c) A particle has total energy E and the force on it is due to potential field $V(x)$. Show that the time taken by the particle to go from x_1 to x_2 is

$$t_2 - t_1 = \sqrt{\frac{M}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$$

assuming the motion to be one-dimensional and that the particle does not reverse its motion.

(d) What is the rotational period of a binary star consisting of two equal mass M and separated by a distance L? [Treat the binary star as a two-body system.]

(e) Show that the trajectory of a particle moving under a central force is confined in a plane.

(f) Angular momentum and angular velocity of a rigid body are not always parallel. Justify the statement.

(g) In streamline flow of Newtonian fluid two streamlines never intersect.— Explain.

2. (a) If $\hat{r}$ and $\hat{\phi}$ denote the unit vectors in plane polar coordinates then,

(i) Prove that $\hat{r}$ and $\hat{\phi}$ forms an orthogonal co-ordinate system.

(ii) Show that $\dot{\hat{r}} = \dot{\phi}\,\hat{\phi}$ and $\dot{\hat{\phi}} = -\dot{\phi}\,\hat{r}$.

(iii) Express the acceleration $\ddot{\vec{r}}(t)$ in the plane polar co-ordinate system.

(b) Set up the equation of motion for a particle falling under constant gravity with a resistive force that is proportional to its velocity. Find out the velocity as a function of time assuming that the body starts from rest. Comment on the motion after a large time. (2+2+2)+(1+2+1)

3. (a) Show that for conservative force $\vec{F}(\vec{r})$ the relation curl $\vec{F}(\vec{r})=0$ holds.

(b) The components of a force in the xy plane are :

$$F_x = x^3 + xy^2$$
$$F_y = y^3 + 3x^2y$$

Compute the work done in a displacement from the point (1, 1) to (2, 2) along the following paths :

(i) a straight line from (1, 1) to (2, 2)

(ii) a straight line from (1, 1) to (1, 2) followed by another straight line from (1, 2) to (2, 2). Could the force field be conservative?

(c) A particle of mass m at rest at $(a, 0, 0)$ is subject to a force $\vec{F}=-\frac{k}{x^3}\hat{i}$ where k is a constant. Find the time taken by the particle to reach the origin. 2+(2+2)+4

4. (a) A frame of reference S' rotates with uniform angular velocity $\vec{\omega}$ with respect to a stationary frame S having common origin. Establish the identity,

$$\frac{d}{dt}=\frac{d'}{dt}+\vec{\omega}\times$$

(b) Hence, obtain expression for centrifugal force and coriolis force for the motion of a particle with respect to the rotating frame.

(c) A particle of mass m is dropped vertically from a height 'h' on north pole. What is the expected deflection of the particle due to coriolis force? 4+4+2

5. (a) Define moment of inertia and principal axis of a rigid body.

(b) Three particles each of mass m are situated at $(a, 0, 0,)$, $(0, a, 0)$ and $(0, 0, a)$. Set up the principal axes of the system and calculate the principal moments of inertia.

(c) A rigid body rotates about an axis having direction cosines (l, m, n) with angular velocity $\vec{\omega}$. Show that the kinetic energy of rotation of the body is, $T=\frac{1}{2}I_{(\mathrm{l,m,n})}\omega^2$, where $I_{\mathrm{l,\,m,\,n}}$ is the moment of inertia of the body about the axis of rotation. 2+4+4

6. (a) Prove that if the centre of mass of a system of particles remains at rest, the total angular momentum of the system of particles about 'any' point is the same and is equal to the angular momentum about the fixed centre of mass.

(b) If the density of the material within a spherical body varies inversely as the distance from the centre, show that the gravitational field inside is the same everywhere. 5+5

7. (a) Set up Euler's equation of hydrodynamics for an incompressible fluid.

(b) Use the relation $\vec{F} = \vec{\nabla} p$ for a fluid at rest (where the symbols have their usual meanings) to establish Archimedes principle.

(c) Water flows through a horizontal tapering tube of circular cross section, the diameter of the entrance and exit ends being 10 cm and 7.5 cm respectively. The pressure difference at two ends is 10 cm of mercury. What is the rate of flow of water through the tube? 4+3+3