2022

PHYSICS — HONOURS

Paper : CC-13

(Syllabus : 2019-2020)

[Digital Systems and Applications]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

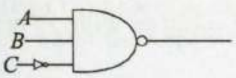
Answer ***question no.*** 1 and ***any four*** from the rest.

1. Answer *any five* questions : 2×5

(a) Convert 45·625 into its binary equivalent.

(b) Subtract $(1011)_2$ from $(1101)_2$ using 2's complement method.

(c) Determine the output expression for the following circuit and simplify it.



(d) Implement the boolean expression $X = AB + \overline{A}C$ using NAND gates only.

(e) What is the basic difference between a S-R Flip-Flop and J-K Flip-Flop?

(f) Design a NOT-gate using a transistor ($\beta_{sat} = 50$) considering $V_{CE\ sat} = 0{\cdot}2V$, $I_C = 5$ mA and source voltage 5 volt.

(g) What is inequality detector?

2. (a) Simplify the following Boolean expression in SOP form using Karnaugh Map.

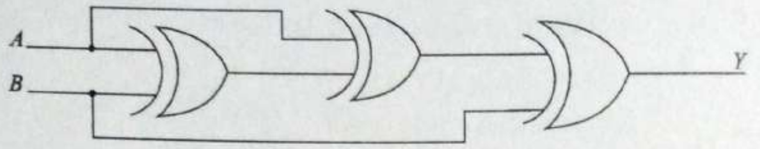
$F(A, B, C, D) = \sum m\,(0, 1, 2, 5, 8, 9, 10)$

(b) Implement the above simplified expression using basic gates.

(c) Make the truth table for the logical function

$f = AB + A\overline{C} + C + AD + A\overline{B}C + ABC$

(d) Write down the Boolean expression for the output (Y) of the following circuit.



3+2+3+2

3. (a) Design a 8 : 1 multiplexer using two 4 : 1 multiplexers.

(b) Implement the following Boolean expression using 8 : 1 multiplexer.

$$F(A, B, C) = \sum m\,(2, 4, 6, 7)$$

How can you use a 8 : 1 multiplexer to implement a logical expression with four inputs?

(c) Write down the basic difference between decoder and de-multiplexer. 3+(3+2)+2

4. (a) Draw the circuit diagram of J-K Flip-Flop and explain its operation using sequence table.

(b) Implement a D-Flip-Flop using J-K Flip-Flop.

(c) Draw the full adder circuit using NAND gate only. (2+3)+2+3

5. (a) What is the basic difference in operation between MS-JK and JK Flip-Flop? Explain with block diagram.

(b) Why is J-K Flip-Flop called an one-bit register? Explain the utility of preset and clear operation in Flip-Flop in this regard.

(c) What is the difference between positive and negative edge triggering? Which type of triggering can be implemented using these triggering? (2+2)+(2+2)+(1+1)

6. (a) What are the differences between Synchronous and Asynchronous counters?

(b) What is shift register? Draw a circuit diagram of a 4-bit shift register.

(c) For 4-bit data transmission, what is the time required if we use SISO and SIPO shift register? Given the duration of each of the clock pulse is 2 ms. 2+(2+3)+3

7. (a) Draw the block diagram of D/A conversion circuit.

(b) A five-bit D/A converter produces $V_{out} = 0{\cdot}2V$ for a digital input of 00001. Find the value of V_{out} for an input 1111.

(c) Design a Mod-10 Asynchronous counter.

(d) Define EPROM. 2+3+4+1

2022

PHYSICS — HONOURS

Paper : CC-14

(Syllabus : 2019-2020)

[Solid State Physics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** 1 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) A crystal cannot diffract uv radiation.— Explain.

(b) Debye temperature of a solid is 1500 K. Compute the highest vibrational frequency of the solid at 30 K.

(c) What is Weiss molecular field?

(d) Distinguish among electronic, ionic and orientational polarisability.

(e) Find the reciprocal lattices of a simple cubic cell.

(f) Explain why the inert gasses do not exhibit paramagnetism.

(g) Perfect diamagnetism and zero resistivity of a superconductor are the two mutually exclusive properties.— Explain.

2. (a) The primitive translation vectors of the hexagonal space lattice are given by :

$\vec{a}_1 = \frac{a}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right), \vec{a}_2 = \frac{a}{2}\left(-\hat{i} + \sqrt{3}\hat{j}\right), \vec{a}_3 = c\hat{k}$. Show that the lattice is its own reciprocal but with a rotation of axes. What is the volume of the unit cell in reciprocal lattice?

(b) Write down Bragg's law. How is this law modified due to refraction at crystal surface?

(c) What are the different symmetry operations of the lattice? (3+1)+(1+3)+2

3. (a) From the Weiss field theory find out the expression of magnetic susceptibility for a ferromagnetic material and show how does it vary above and below the Curie temperature.

(b) The refractive index and the dielectric constant of water are 1·33 and 81 respectively. Determine the percentage of ionic polarisability. (3+2+2)+3

4. (a) Obtain the dispersion relation for one dimensional monoatomic lattice.

(b) In the Krönig-Penny model, the following equation is obtained after simplification.

$$\frac{P}{\alpha a}\sin\alpha a+\cos\alpha a=\cos k a \quad \text{where } \alpha=\sqrt{\frac{2mE}{\hbar^2}}$$

(i) What is the physical significance of P?

(ii) By plotting the above equation as a function of $2a$, discuss the allowed and forbidden regions of energy from the plot (Assume a fixed value of P).

(c) Derive the Wiedemann-Franz law and what is the most significant conclusion that we arrive from this law? 3+(1+3)+3

5. (a) Establish the relationship among the electric displacement $(\vec{D})$ field strength $(\vec{E})$ and polarisation vectors $(\vec{P})$.

(b) Explain the terms : electronic polarisation and electronic polarisability.

(c) Silicon has the dielectric constant 12 and the edge length of the conventional cubic cell of silicon lattice is 5.43Å. Calculate the electronic polarisability of silicon atoms. (silicon has diamond crystal structure i.e., two interpenetrating FCC shifted along diagonal by $^1/_4$th of it). 3+2+5

6. (a) Calculate the Hall coefficient in a solid where both the electrons and holes contribute to the carrier flow. Can the Hall coefficient be zero? Explain.

(b) Distinguish between type I and type II superconductors with the help of M-H plot.

(c) The magnetic field strength in a piece of metal is 10^6 amp/m. Find the flux density and magnetisation of the material. (Assume that the magnetic susceptibility of the metal is 0.5×10^{-5}) (3+1)+2+4

7. (a) Explain the necessity of the concept of effective mass in discussing the behaviour of an electron in periodic lattice. Is the effective mass equal to the true mass of a free elcotrm?

(b) Obtain the expression of specific heat of solid according to Einstein's theory. Also mention its form in both high and low temperature limit. Why is the Debye theory of specific heat more acceptable than Einstein's theory? (2+1)+(4+2+1)

2022

PHYSICS — HONOURS

Paper : DSE-A2

[(a) Nanomaterials and Applications]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer *any five* questions : 2×5

(a) How does band gap of a semiconductor change as it is reduced from bulk dimension to nanodimension?

(b) Use Scherrer equation to calculate size of nanocrystallite, when diffraction peak is obtained at $\theta = 30°$ having FWHM : 0.8, using radiation of wavelength $\lambda = 0.154$nm (Scherrer constant = 0.9).

(c) Distinguish between shallow and deep level trap states.

(d) Determine density of states of infinitely deep potential well of width 1nm corresponding to energy E = 3eV.

(e) Quantum dots are considered as artificial atoms. — Explain.

(f) State basic principle of NEMS.

(g) Discuss use of ball milling technique in ceramic processing very briefly.

Group – B

2. Answer *any three* questions : 5×3

(a) Briefly describe Chemical Vapour Deposition (CVD) process for synthesis of nanomaterials.

(b) What is the effect of size quantization on the emission and absorption spectra of a material?

(c) Compare SEM and TEM for the study of surface of thin film with brief description of operational difference. Also, state the effectivity of the techniques to study distribution of nanoparticles.

(d) Show that quantized conductivity in 1-D channel is $2e^2/h$.

(e) Why are K-selection rules relaxed in a nanosystem? What is its effect on the probability of electron transition?

Group – C

Answer *any four* questions.

3. Define density of states. Derive its expression corresponding to quantum well (2D) and quantum wire (1D). Represent it graphically in the same plot for both the systems. 2+(3+3)+2

4. (a) Distinguish between exciton and polaron. Explain why excitons do not contribute to electrical conduction. State the conditions to observe exciton.

   (b) Distinguish between direct tunnelling and trap-assisted tunnelling. State temperature dependence of the two processes. {(2+2)+2}+(2+2)

5. (a) Give brief outline of sol-gel method for the synthesis of nanoparticles. Discuss its limitations.

   (b) Describe the basic working principle of Molecular Beam Epitaxy (MBE). (4+2)+4

6. (a) What do you understand by Coulomb blockade effect? State the conditions to observe it.

   (b) Calculate the maximum temperature at which Coulomb blockade is observed in a tunnel junction having C = 1pF. Also, estimate the maximum dimension of the device to observe Coulomb blockade at room temperature (300K). (2+3)+(3+2)

7. (a) Give qualitative description of single electron transfer device (SED). Mention some applications.

   (b) How can photoluminescence (PL) be used to characterize nanomaterials and briefly mention the technique to record the response. (3+2)+(3+2)

8. Write short notes on : 5+5

   (a) Photolithography for nanofabrication

   (d) CNT (Carbon nanotube) based transistors.

Paper : DSE-A 2

[(b) Advanced Classical Dynamics]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any five* questions : 2×5

(a) Establish the symmetry that leads to the conservation of angular momentum in Lagrangian formalism.

(b) Calculate the Poisson brackets $\{x, L_z\}$ and $\{y, L_z\}$, where L_z is the z-component of the angular momentum of a particle.

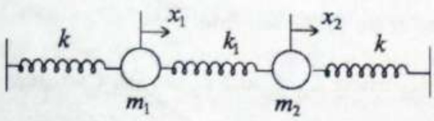
(c) Show that for an inverse square field the virial theorem takes the form $2\langle T\rangle + \langle V\rangle = 0$, where the symbols have their usual meanings.

(d) What do you mean by a dissipative dynamical system? Explain with an example.

(e) Find the nature of the fixed point $x = 0$ in case of the dynamical system represented by $\dot{x} = \sin x$.

(f) A particle of mass 'm' moves with constant velocity in a straight line. Justify if it has an angular momentum.

(g)



For the above system, set up the Lagrangian.

Group - B

2. Answer *any three* questions : 5×3

(a) Show that the geodesics on a circular cylinder are helices as $az + b\theta$ = constant.

(b) A flexible chain of fixed length is suspended between two fixed points. Find the curve that will minimize the gravitational potential energy of the system.

(c) What is a conservative dynamical system? An oscillator is described by the equation

$$\ddot{x} - k(1 - x^2)\dot{x} + \omega^2 x = 0$$

Convert it to a dynamical system and check whether it is dissipative.

(d) If a Lagrangian $L(q, \dot{q})$ has no explicit dependence on time, prove that $\left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right)$ is a constant of motion.

(e) A bead of mass 'm', suspended by an extensible string from a rigid support is executing small oscillations. Use Lagrange multiplier to determine the tension in the string at an angle θ from the vertical, using Lagrangian formulation.

Group - C

Answer ***any four*** questions.

3. (a) The Lagrangian of a spherical pendulum is given by $L = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta\right) - mga\cos\theta$, where '$\theta$' and '$\varphi$' represent the spherical polar coordinates of the bob of mass 'm' on a sphere of radius 'a'. Find the canonical momenta conjugate to 'θ' and 'φ' and obtain the Hamiltonian of the system.

(b) The Lagrangian of a particle of charge 'q' and mass 'm' moving under a magnetic field characterized by a time-independent vector potential '$\vec{A}$' is given by $L = \frac{1}{2}mv^2 - q\,\vec{v}.\vec{A}$ Derive the Lorentz force equation from this. (2+4)+4

4. (a) Find the conditions on the real parameters α, β, γ and δ, such that $\dot{q} = \alpha q + \beta p$ and $\dot{p} = \gamma q + \delta p$ are Hamilton's equation of motion for some $H(p, q)$. Hence find $\frac{dH}{dt}$.

(b) Show that the Poisson bracket $\{L_x, L_y\} = L_z$, where L_x, L_y and L_z are the Cartesian components of the angular momentum $\bar{L}$.

Hence show that $\{L_x, \{L_y, L_z\}\} + \{L_y, \{L_z, L_x\}\} + \{L_z, \{L_x, L_y\}\} = 0$ (3+2)+(2+3)

5. (a) Find the planar curve C having the given perimeter L, that encloses the largest area.

(b) The moment and product moment of inertia of a rigid body with respect to a Cartesian coordinate system are I_{xx}, I_{yy}, I_{zz} and I_{xy}, I_{yz}, I_{zx} respectively. Prove that the moment of inertia of the rigid body about an axis making angles α, β and γ with the x, y and z axes respectively is given by
$I = I_{xx}\cos^2\alpha + I_{yy}\cos^2\beta + I_{zz}\cos^2\gamma + 2I_{xy}\cos\alpha\cos\beta + 2I_{yz}\cos\beta\cos\gamma + 2I_{zx}\cos\gamma\cos\alpha$. 4+6

6. (a) Consider a linear triatomic molecule with two outer atoms of mass 'm' connected to the central atom of mass 'M' by two identical massless springs of spring constant 's'. Show that the normal frequencies of small oscillations are '0' $\sqrt{\frac{s}{m}}$ and $\sqrt{\frac{s}{m} + \frac{2s}{M}}$.

(b) Consider a forced linear harmonic oscillator given by

$$m\ddot{x} + m\omega^2 x = \tau f\,\delta\,(t - t_0)$$

where τ is a quantity having a dimension of time. Discuss qualitative the behaviour of $x(t)$ and $\dot{x}(t)$ immediately before and after $t = t_0$. (2+2+2)+(1+1+1+1)

7. (a) Find out the fixed points of the 1D dynamical system described by

$$\dot{x} = x - x^3$$

Draw the phase portrait and the flow of x. Hence ascertain the nature of the fixed points.

(b) Consider the 2D dynamical system

$$\dot{x} = y$$

$$\dot{y} = x$$

(i) Write down the fixed point and the stability matrix.

(ii) Determine the eigenvalues of the stability matrix and hence conclude about the nature of the fixed point.

(iii) Find the eigendirections and draw the flow lines with respect to these directions. (1½+2+1½)+(1+2+2)

8. (a) Consider the map $x_{n+1} = \cos x_n$ that has a fixed point at $x^* \approx 0.74$. Check analytically that the fixed point is stable. Show graphically how the stable point is approached if one starts from a nearby point.

(b) Consider the 1D dynamical system given by $\dot{x} = rx - x^2$.

(i) Draw the phase portraits for $r > 0$, $r = 0$ and $r < 0$.

(ii) From the phase portrait and the corresponding flows determine the nature of the fixed points in the 3 cases. (2+2)+{(1+1+1)+(1+1+1)}

2022

PHYSICS — HONOURS

Paper : DSE-B2

[(a) Communication Electronics]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) In an FM system, the frequency deviation is about 20·5 kHz and a modulating signal frequency is 5 kHz. Determine the modulation index and carrier swing.

(b) Assume the message signal $x(t) = 15\cos(2\pi t)$ volts and carrier wave $c(t) = 45\cos(100\pi t)$ volts. Derive AM wave for 30% modulation.

(c) What is sampling? State sampling theorem.

(d) What is multiplexing? Name the types of multiplexing.

(e) What is demodulation?

(f) Explain the following terms in connection with satellite communication :

(i) geostationary satellite,

(ii) uplink and downlink frequencies.

(g) What are SIM and IMEI number in mobile communication?

Group - B

2. Answer ***any three*** questions :

(a) (i) Show that the total power of a fully amplitude modulated wave is 1·5 times the unmodulated carrier power.

(ii) Show that the AM wave can be represented by a carrier and two side bands. 3+2

(b) How can you design an amplitude modulator by using an amplifier whose input v_i and output (v_o) characteristics is $v_o = a_1 v_i + a_2 v_i^2$? where a_1 and a_2 are constants. 5

(c) (i) How is digital modulation different from analog modulation?

(ii) Describe amplitude shift keying (ASK).

(iii) Define bit rate. 2+2+1

Please Turn Over

(d) (i) Find the Nyquist rate for the signal $x(t) = 25 \cos(500\pi t)$.

(ii) Find the bandwidth of 8-PSK.

(iii) The upper and lower cut-off frequencies of a resonant circuit are found to be 8·07 MHz and 7·93 MHz respectively. Calculate the bandwidth. 2+2+1

(e) What do you mean by transponder in satellite communication? What are their basic components? 3+2

Group - C

Answer ***any four*** questions.

3. An audio signal : $15 \sin 2\pi(1500t)$

Amplitude modulates a carrier : $60 \sin 2\pi(1000000t)$.

(a) Sketch the audio signal.

(b) Sketch the carrier.

(c) Construct the modulated wave.

(d) Determine the modulation factor and percentage modulation.

(e) What are the frequencies of the audio signal and carrier?

(f) What frequencies would show up in a spectrum analysis of the modulated wave? 1+1+2+2+2+2

4. (a) Find the expression of frequency modulated (FM) wave.

(b) A 80 MHz carrier is frequency modulated, the modulation index being 4. The frequency of information signal is 10 kHz. What is the maximum frequency deviation?

(c) What do you mean by resistor noise? Calculate the thermal noise voltage developed in a resistor $R = 100\ \Omega$. The bandwidth of the circuit is 5 kHz at room temperature 30°C.
(Given $k_B = 1{\cdot}38\times10^{-23}$ J/K) 3+3+(2+2)

5. (a) Draw the circuit diagram for generation of PAM signal and explain its operation.

(b) Draw the block diagram of PAM signal reception.

(c) Draw the circuit diagram of a zero order holding circuit. (3+3)+2+2

6. (a) Define μ-law for companding. Define unipolar RZ and NRZ.

(b) What is constellation diagram? Draw the diagram for 8–PSK.

(c) How can non-uniform quantization be used to increase SNR? (2+2)+(2+2)+2

7. (a) What is path loss of satellite communication system? How is the path loss related to the gain and power of the transmitting and receiving antenna?

(b) In satellite communication $P_t = 23\ dB_m$, $G_t = 2dB_i$, $G_r = 2dB_i$, $P_r = -71\ dB_m$. Find the path loss. Where P_t = Power of a transmitter, G_t = Gain of a transmitter, P_r = Power of a receiver, G_t = Gain of a receives.

(c) Draw the block diagram of Earth station. (2+3)+3+2

8. (a) What is Carson's rule of thumb for the determination of bandwidth in FM station?

(b) Describe the basic principle of satellite communication.

(c) What are the differences among 2G, 3G and 4G technologies in mobile communication system? 3+4+3

———————

Paper : DSE-B-2

[(b) Advanced Statistical Mechanics]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer *any five* questions : 2×5

(a) Explain the following statement — 'Negative temperatures are hotter than positive temperatures'.

(b) A three-level single particle system has five microstates with energy 0, ε, ε, ε, and 2ε. What will be the mean energy of the system if it is in equilibrium with a heat bath at temperature T?

(c) Give a schematic diagram of chemical potential (μ) vs. temperature (T) curves for two different systems — the ideal Bose gas and the ideal Fermi gas.

(d) Three identical spin 1/2 particles of mass m are free to move within a one-dimensional rigid box of length L. Assuming that they are non-interacting, find the energies of the two lowest energy eigenstates in units of $\frac{\pi^2\hbar^2}{2mL^2}$.

(e) In a thermodynamic system in equilibrium, each particle can exist in three possible states with probabilities 1/2, 1/3, and 1/6 respectively. Find the per particle entropy of the system.

(f) What is 'Chandrasekhar mass limit'?

(g) Draw the specific heat curve for a Bose gas as a function of temperature (T) on both sides of critical temperature (T_c).

Group – B

2. Answer *any three* questions : 5×3

(a) Consider a classical gas of N identical indistinguishable particles in a two-dimensional square box of side L. If the total energy of the gas is E, find the number of accessible microstates and the entropy.

(b) Calculate the pressure exerted by an ideal Fermi gas at 0 K. What is the physical reason for the non-zero pressure at absolute zero?

(c) The wave function $\psi(t)$ of an isolated system is given by $\psi = \sum_n a_n(t)\varphi_n$ where $\{\varphi_n\}$ is the complete orthonormal set of stationary wave functions. Write down the postulate of equal a priori probabilities and the random phases in terms of a_n.

(d) A particle hops on a one-dimensional lattice with lattice spacing a. The probability of the particle to the neighbouring site to its right is p, while the corresponding probability to hop to the left is q = 1–p. Find the root-mean-squared deviation $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ in displacement after N steps.

(e) Use canonical ensemble to prove that specific heat (C_v) at thermal equilibrium cannot be negative.

Group – C

Answer ***any four*** questions.

3. (a) Show that the relative root-mean-squared fluctuation in energy of an N-particle system in contact with a heat reservoir varies as $1/\sqrt{N}$. Hence comment on the equivalence of the canonical ensemble to the microcanonical ensemble in thermodynamic limit.

(b) An ideal collection of N two-level systems is in thermal equilibrium at temperature T. Each system has a ground state energy $-\varepsilon$ and an excited state energy $+\varepsilon$. Prove that the Helmholtz free energy of the system is $A = -Nk_BT\ln\left\{2\cosh\left(\frac{\varepsilon}{k_BT}\right)\right\}$. (5+2)+3

4. (a) Write down the grand partition function of an ideal Bose gas of fugacity z, volume V and temperature T.

(b) The number of Bosons in the excited states can be expressed as $N_e = N - N_0 = \frac{V}{\lambda^3} g_{\frac{3}{2}}(z)$, where $\lambda = h/\sqrt{2\pi m k_B T}$ and $g_{\frac{3}{2}}(z)$ is the monotonically increasing Bose function. Given that the largest value (≈ 2.612) of $g_{\frac{3}{2}}(z)$ is bounded at $z = 1$, derive the condition to obtain Bose-Einstein condensate.

(c) Show that in the condensed phase ($T < T_C$), $N = N_0 + N\left(\frac{T}{T_C}\right)^{3/2}$. 2+5+3

5. Consider the ionization of atomic hydrogen into a hydrogen ion and an electron : $H \rightleftharpoons H^+ + e^-$. The number densities of the neutral hydrogen atoms, the hydrogen ions and the electrons at temperature T are n_H, n_{H^+}, and n_e respectively.

(a) Ignoring the excited bound states derive the Saha ionisation equation

$$\frac{n_H + n_e}{n_H} = \frac{g_H + g_e}{g_H} \frac{\lambda_H^3}{\lambda_{H^+}^3 \lambda_e^3} e^{-I/k_BT}$$

where the gs represent the statistical weights of the three species — the neutral hydrogen atoms, the hydrogen ions and the electrons. The λs represent their thermal wavelengths $\left(\lambda = h/\sqrt{2\pi m k_B T}\right)$ and I is the ionization energy of a hydrogen atom.

(b) Since, $g_H = g_e = 2$, $g_{H^+} = 1$, $m_{H^+} \approx m_H$ and $n_H = n_e$ (overall charge neutrality) show that we may write Saha's equation as $\frac{x^2}{1-x} = \frac{1}{n_e \lambda_e^3} e^{-1/k_B T}$, where $x = \frac{n_{H^+}}{\left(n_H + n_{H^+}\right)}$ is the fraction of hydrogen atoms that are ionised.

(c) At the surface of the sun the temperature is about 5800 K and the number density of electrons is $2 \times 10^{19}\ m^{-3}$. Using the Saha's equation in (b) show that less than one hydrogen atom in 10000 is ionised. 5+3+2

6. (a) The energy eigenvalues of a one-dimensional harmonic oscillator are given by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, n = 0, 1, 2, ...$$

Find the internal energy of a system of N such independent harmonic oscillators in thermal equilibrium at temperature T. Calculate $C_P - C_V$ for this system.

(b) The Hamiltonian of a classical oscillator in two-dimension in plane polar coordinate is

$$H = \frac{p_r^{\ 2}}{2m} + \frac{p_\theta^{\ 2}}{2mr^2} + \frac{1}{2}kr^2$$

where the symbols have their usual meaning.
Calculate $\langle H \rangle$. You may use the generalised equipartition theorem. (4+2)+4

7. (a) Consider the Hamiltonian for the Ising model with N spins

$$H = -J\sum_{\langle ij \rangle} s_i s_j$$

with $s_i = \pm 1$, $J > 0$ and $\sum_{\langle ij \rangle}$ is a sum over nearest neighbours. Within Bragg Williams approximation the average magnetization per spin (m) can be expressed by the following relation :

$$m = \tanh(J\gamma m/k_B T),$$

where γ is the number of nearest neighbours. Use this relation to show that there exists a critical temperature $T_C = J\gamma/k_B$ below which the system can have a non-zero spontaneous magnetization and above cannot.

(b) Calculate the entropy $S = -k_B T_r(\rho ln \rho)$ for the following density matrix $\rho = \begin{bmatrix} \tau - 1 & 0 \\ 0 & \tau + 1 \end{bmatrix}$, where τ is a real parameter and the rest of the symbols have usual meaning.

(c) For a system having $V^{2/3} E =$ constant, calculate the pressure, P as a function of energy, E and volume, V. Hence find the relation between P, V and E. 4+3+3

8. (a) Consider the degeneracy parameter $e^{\alpha} = e^{-\left(\frac{E_F}{kT}\right)}$ of FD gas. Now, depending on this temperature how would you classify the degenerate state of FD gas? (whether it is very weakly degenerate, weakly degenerate, degenerate or strongly degenerate?)

(b) Show that the number of Fermions per unit volume of a strongly degenerate FD gas is

$$n = \left(\frac{8\pi}{3}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2} F_{F_0}{}^{3/2}.$$

(c) What is Fermi temperature? Show that the degeneracy parameter $p_0 = \frac{2}{5} n k_B T_F$, where n is the number of molecules per unit volume and T_F is the Fermi temperature. (2+4)+1+3

———