# 2023

## PHYSICS — HONOURS

**Paper : CC-7**

**(Modern Physics)**

**Full Marks : 50**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question no.*** 1 and ***any four*** questions from the rest.

2×5

1. Answer ***any five*** questions :

   (a) Show that the operator $\frac{d}{dx}$ is not Hermitian.

   (b) Find the eigenfunction of $x+\frac{d}{dx}$.

   (c) Show that if $\hat{H}$ is a Hermitian operator, then $e^{i\hat{H}}$ is unitary.

   (d) State at least two implications of $[H, L] = 0$, where $H$ and $L$ are Hamiltonian and angular momentum operators respectively.

   (e) Find the ground state spin and parity of $^{27}_{13}\text{Al}$.

   (f) Explain why closed-shell nuclei must be spherically symmetric.

   (g) What are metastable states?

2. (a) Draw the spectral distribution of black body radiation as a function of wavelength for two different temperatures.

   (b) The photoelectric work-function for Li is 2.3 eV. Find the threshold wavelength for photoelectric effect. If ultraviolet light of wavelength 2000 Å is incident on Li surface, obtain the values of maximum kinetic energy of photoelectron and stopping potential.

   (c) Calculate the value of Compton wavelength of an electron.

   (d) Find the de Broglie wavelength of an electron, accelerated through a potential difference of 100 V.

2+3+2+3

3. (a) A triangular hat wave function is given by

$$\psi(x) = \frac{Ax}{a}, 0 \le x \le a$$
$$= \frac{A(b-x)}{(b-a)}, a \le x \le b$$
$$= 0, \text{ otherwise}$$

where $A$, $a$ and $b$ are constants.

(i) Sketch the wave function.

(ii) Determine the normalization constant $A$ in terms of $a$ and $b$.

(iii) Calculate $\langle x \rangle$.

(b) Show that the momentum operator $p_x = -i\hbar\frac{d}{dx}$ for a free particle moving in one dimension does not have normalizable eigenfunctions.

(c) Show that $[x, p_x] = i\hbar$. (1+2+3)+2+2

4. (a) If $\rho(\mathbf{r}, t) = \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)$ is a probability density, then show that there must be a probability current $\vec{J}(\mathbf{r}, t) = \frac{i\hbar}{2m}\left(\psi^* \nabla\psi - (\nabla\psi^*)\psi\right)$ that satisfies the equation

$$\frac{\partial}{\partial t}\rho(\mathbf{r}, t) + \vec{\nabla}.\vec{J}(\mathbf{r}, t) = 0 .$$

(b) Derive an expression for the time derivative of the expectation value of momentum in one dimension, that is for $\frac{d\langle \hat{p}_x \rangle}{dt}$. 6+4

5. (a) What are the difficulties faced in explaining the observed β-ray energy spectrum? How were these difficulties removed with the help of Pauli's neutrino hypothesis?

(b) Is electron positron pair production possible in vacuum? — Explain.

(c) Is the nuclear force between two protons different from that between a proton and a neutron? What do we normally attribute the difference to? (2+2)+2+4

6. (a) For the fission

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$

calculate the $Q$-value.

[Mass of $^{235}U$ = 235.04278 u;
$^{1}n$ = 1.00866 u;
$^{141}Ba$ = 140.9129 u;
$^{92}Kr$ = 91.89719 u ]

(b) $^{7}_{4}Be$ and $^{7}_{3}Li$ have atomic masses 7.01693 u and 7.01600 u respectively. Which of them shows $\beta$ activity and of what type? Justify your answer.

(c) Give one example of $\beta^-$-decay, $\beta^+$-decay and electron capture. 4+3+3

7. (a) Explain spontaneous and stimulated emissions.

(b) Draw the energy level diagrams of three-level and four-level lasers and explain the advantages of four-level laser over three-level laser.

(c) Explain the working principle of Ruby laser with suitable diagram. Write the disadvantage of Ruby laser. (2+2)+2+(3+1)