2024

MATHEMATICS — HONOURS

Paper : CC-11

(Probability and Statistics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative with proper justification (wherever applicable) : 2×10

(a) A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If $P(B)=\frac{3}{2}P(A)$ and $P(C)=\frac{1}{2}P(B)$, then $P(A)$ is given by

(i) $\frac{5}{13}$ (ii) $\frac{4}{13}$

(iii) $\frac{3}{13}$ (iv) $\frac{2}{13}$.

(b) 6 boys and 6 girls sit in a row at random. Then the probability that 6 girls sit together is

(i) $\frac{1}{132}$ (ii) $\frac{1}{32}$

(iii) $\frac{1}{360}$ (iv) $\frac{1}{36}$.

(c) If X is a Poisson random variate with mean 2, then $P(|X-3|<1)$ will be

(i) $\frac{3e^{-2}}{2}$ (ii) $3e^{-2}$

(iii) $\frac{4e^{-2}}{3}$ (iv) $\frac{3e^{-2}}{4}$.

(d) The median of a distribution with density function $f(x)=e^{-x}$, $x>0$ is

(i) $\log(2)$ (ii) 2

(iii) $\log\left(\frac{1}{2}\right)$ (iv) $\log\left(\frac{1}{3}\right)$.

(e) The characteristic function of Poisson distribution with parameter m is

(i) $e^{m(it-1)}$ (ii) $e^{m(e^{it}-1)}$

(iii) e^{mit} (iv) $e^{(mit-1)}$.

(f) If the joint p.d.f. of (X, Y) is given by $f(x, y)=\begin{cases} e^{-y}, & 0<x<y<\infty \\ 0, & \text{otherwise} \end{cases}$, then $f_x(x|y)$ is

(i) $\dfrac{1}{e^y-1}$ (ii) $\dfrac{e^y}{y}$

(iii) $\dfrac{e^y}{e^y-1}$ (iv) $\dfrac{1}{y}$.

(g) If $E(XY)=3$ and $E(X)=2=E(Y)$, then the covariance of the random variables $(2X+10)$ and $\left(\dfrac{-5}{2}Y+3\right)$ is

(i) 3 (ii) 5

(iii) 2 (iv) –5.

(h) The first and second moments of a probability distribution about the point 2 are 1 and 16, respectively. Then the mean and variance are respectively

(i) 1, 15 (ii) 3, 12

(iii) 3, 15 (iv) 2, 16.

(i) If T_1, T_2 be statistic with expectation $E(T_1)=2\theta_1+3\theta_2$ and $E(T_2)=\theta_1+\theta_2$, then the unbiased estimate of the parameter θ_1 is

(i) $3T_2-T_1$ (ii) $3T_2+2T_1$

(iii) $2T_2-3T_1$ (iv) $3T_2T_1$.

(j) If $x\geq 1$ is the critical region for testing $H_0:\theta=2$ against the alternative hypothesis $H_1:\theta=1$, then on the basis of the single observation from the population $f(x, \theta)=\theta e^{-\theta x}$, $0\leq x<\infty$ the type-II error is

(i) $\dfrac{1}{e^2}$ (ii) $\dfrac{1}{(1-e)}$

(iii) $\dfrac{(e-1)}{e}$ (iv) e^2.

Unit - 1

Answer *any two* questions.

2. Define a σ-field and probability function. Two dice are tossed. Find the probability of getting 'an even number on the first die or a total of 8'. 3+2

3. Show that the expectation of the number of failures preceding the first success in an infinite sequence of Bernoulli trials with probability of success p is $(1-p)/p$. 5

4. If $X \sim N(m, \sigma)$, then prove that $\mu_{2K+2} = \sigma^2\mu_{2K} + \sigma^3 \frac{d\mu_{2K}}{d\sigma}$. Hence find β_2 (coefficient of Kurtosis). 3+2

Unit - 2

Answer *any two* questions.

5. The joint probability density function of two random variates X, Y is given by

$$f(x,y) = \begin{cases} \dfrac{6-x-y}{8}, & 0 < x < 2,\ 2 < y < 4 \\ 0, & \text{elsewhere} \end{cases},$$

Find $P(X+Y<3)$ and $P(X<1 \mid Y<3)$. 5

6. Let X and Y be any two random variables. Then prove that $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$.

If $\text{Var}(X) = 4$, $\text{Var}(Y) = 9$ and $\text{Var}(X-Y) = 16$, then find the value of $\text{cov}(X, Y)$. 3+2

7. If θ be the acute angle between two regression lines, then prove that

$$\tan\theta = \frac{1-\rho^2}{\rho}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2},$$

where σ_x, σ_y are the standard deviations of X and Y respectively and ρ is the correlation coefficient. Interpret the cases when $\rho = 0$ and $\rho = 1$. 5

Unit - 3

Answer *any one* question.

8. Use Tchebycheff's inequality to show that for $n \geq 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{1}{6}n - \sqrt{n}$ and $\frac{1}{6}n + \sqrt{n}$ is at least $\frac{31}{36}$. 5

9. Use De Moivre-Laplace limit theorem to find the number of times a die has to be thrown such that the probability that the absolute difference between the frequency ratio of sixes and $\frac{1}{6}$ is less than 0.01, is 0.99. $\left(\text{Given}: \frac{1}{\sqrt{2\pi}} \int_{2.58}^{\infty} e^{-\frac{x^2}{2}} dx = 0.005\right)$ 5

Unit - 4

Answer *any two* questions.

10. The variable X is normally distributed with mean 68 cm and standard deviation 2.5 cm. What should be the size of the sample whose mean shall not differ from the population mean by more than 1 cm with probability 0.95?

[Given that the area under standard normal curve to the right of the ordinate at 1.96 is 0.025] 5

11. Let $X_1, X_2, \ldots, X_n$ be a set of n mutually independent variates, each normal, then show that the statistic

$$t = \frac{\sqrt{n}\left(\bar{X} - m\right)}{s}$$

is t-distributed with $\gamma = (n-1)$ degrees of freedom, where m denotes the population mean and $\bar{X}$ and s are respectively the sample mean and unbiased estimate of sample standard deviation. 5

12. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Obtain the 98% confidence limits for the percentage of bad apples in the consignment.

$$\left(\text{Given}: \int_{2.33}^{\infty} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = 0.01\right)$$ 5

13. 'A' tossed a biased coin 50 times and got head 20 times while 'B' tossed it 90 times and got 40 heads. Find the maximum likelihood estimate of the probability of getting head when the coin is tossed. 5

Unit - 5

Answer *any two* questions.

14. Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type-I error and the power of the test.

3+2

15. State Neyman-Pearson lemma and relate it with the idea of critical region. 3+2

16. Nine patients to whom a certain drug was administered, registered following rise in blood pressure :

$$3, 7, 4, -1, -3, 6, -4, 1, 5$$

Test the hypothesis that the drug does not raise blood pressure at 10% level of significance, assuming normal population (Given : $P(t > 1.86) = 0.05$ for 8 degrees of freedom). 5

17. Fit a straight line of the form $y = ax + b$ to the following data by the method of least square : 5

x	15	20	25	30	35
y	12	14	18	25	31