2024

MATHEMATICS — HONOURS

Paper : DSE-B-1.1, DSE-B-1.2 and DSE-B-1.3

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Paper : DSE-B-1.1

(Discrete Mathematics)

Full Marks : 65

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any (**1** mark for correct option and **1** mark for justification) : 2×10

(a) What is the maximum number of possible non-zero values in an adjacency matrix of a simple graph with n vertices?

(i) $n^2 - n$ (ii) $\frac{n(n+1)}{2}$

(iii) $\frac{n(n-1)}{2}$ (iv) n.

(b) For a given graph G having n vertices and e edges which is connected and has no cycles, which of the following statements is true?

(i) $n = e$ (ii) $n = e + 1$

(iii) $n = e - 1$ (iv) $n = e + 2$.

(c) A graph with n vertices will definitely have a parallel edge or self-loop if the total number of edges are

(i) greater than $n - 1$ (ii) less than $n(n - 1)$

(iii) greater than $\frac{n(n-1)}{2}$ (iv) less than $\frac{n^2}{2}$.

(d) What is the possible degree sequence of a graph with 5 vertices and 5 edges?

(i) 3, 4, 2, 2, 2 (ii) 6, 6, 5, 3, 2

(iii) 3, 2, 2, 2, 1 (iv) 4, 2, 2, 2, 2.

(e) Which of the following is true about a Hamiltonian cycle?

(i) Includes all the edges

(ii) Includes all the vertices

(iii) Includes a regular graph as an induced subgraph

(iv) All vertices are of odd degree.

(f) For $n \geq 1$, and positive integers a, b, if $gcd(a, b) = 1$, then $gcd(a^n, b^n)$ is

(i) n (ii) na

(iii) nb (iv) 1.

(g) Which one is a perfect number of the following numbers?

(i) 21 (ii) 25

(iii) 28 (iv) 30.

(h) The order of U_{20} is

(i) 2 (ii) 4

(iii) 6 (iv) 8.

(i) Value of $\phi(1024)$ is

(i) 2^5 (ii) 2^8

(iii) 2^9 (iv) 2^{10}.

(j) Which of the following integers has a primitive root?

(i) 15 (ii) 16

(iii) 17 (iv) 18.

Unit - I

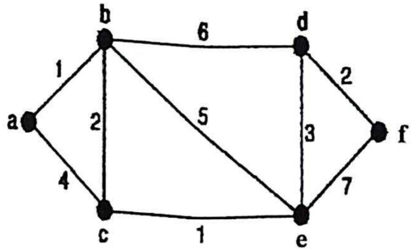
Answer *any five* questions.

2. If $m > \frac{(n-1)(n-2)}{2}$ in a simple graph G with n vertices and m edges, then prove that G is connected. 5

3. If G is a connected simple graph with no cycles, prove that G has at least two pendant vertices. 5

4. Prove that the graph $K_{3,3}$ is not a planar graph. Also prove if we delete any vertex from it, then it becomes a planar graph. 3+2

5. Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to f. 5



6. Prove that a tree with n vertices has $n-1$ edges. 5

7. Prove that a simple connected graph G is Eulerian if and only if degree of each vertex of G is even. 5

8. For the poset $(P, \subseteq)$, where $P = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, determine (a) the greatest element, (b) the least element, (c) all maximal elements (d) all minimal elements. 1+1+1+2

9. Give an example of a graph which is both Eulerian and Hamiltonian. 5

Unit - II

Answer ***any four*** questions. 5×4

10. Find the remainder when $(n^2 + n + 41)^2$ is divided by 12.

11. Show that if p is a positive integer such that both p and $p^2 + 2$ are prime, then $p = 3$.

12. If the difference between two positive integers is n, then prove that their *gcd* divides n.

13. Prove that $3a^2 - 1$ is never a perfect square.

14. If ϕ is Euler-Totient function, then prove that $\phi(p^a) = p^a\left(1-\frac{1}{p}\right)$ for any prime p and $gcd(a, p) = 1$.

15. Show that the cube of any natural number is of the form 7K or 7K $\pm$ 1.

16. If p be prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.

Please Turn Over

Paper : DSE-B-1.2

(Linear Programming and Game Theory)

Full Marks : 65

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any. (**1 mark** for correct option and **1 mark** for justification.) 2×10

(a) Suppose that the objective function of an L.P.P. assumes its optimal value at more than one extreme point. Then

(i) the value of the objective function will be different for different convex combinations of these extreme points.

(ii) the convex combination of these extreme points will improve the value of the objective function.

(iii) it indicates that the number of basic feasible solutions is degenerate.

(iv) every convex combination of these extreme points also gives the optimal value of the objective function.

(b) Consider two sets $X = \{(x_1, x_2) \mid 2x_1 + x_2 \leqslant 4,\ x_1 + 4x_2 \geqslant 7,\ x_1, x_2 \geqslant 0\}$ and $Y = \{y \mid |y| \leqslant 3\}$. Then

(i) X is a convex set, but Y is not a convex set

(ii) X is not a convex set, but Y is a convex set

(iii) none of X and Y are convex sets

(iv) both X and Y are convex sets.

(c) The following L.P.P :

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 1 \\ & -3x_1 + x_2 \ge 3 \\ & x_1 \ge 0,\ x_2 \ge 0 \end{aligned}$$

has

(i) one feasible solution

(ii) two feasible solutions

(iii) infinite number of feasible solutions

(iv) no feasible solution.

(d) For the system of equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + x_2 + 5x_3 = 9$$

the solution $x_1 = 5$, $x_2 = -1$, $x_3 = 0$ is

(i) basic and feasible
(ii) feasible but not basic
(iii) degenerate basic
(iv) non-degenerate basic.

(e) If the primal problem has m constraints with n variables, then the dual problem has

(i) m constraints with n variables
(ii) n constraints with m variables
(iii) $m + n$ constraints with mn variables
(iv) mn constraints with $m + n$ variables.

(f) In a balanced transportation problem with m sources and n destinations, the number of dual variables will be

(i) $m + n$
(ii) $m + n - 1$
(iii) $m + n + 1$
(iv) mn.

(g) In a travelling salesman problem, the salesman has to visit n cities. Then the number of possible routes are

(i) n
(ii) $n - 1$
(iii) $\lfloor n$
(iv) $\lfloor n-1$.

(h) The initial basic feasible solution of the transportation problem using VAM is

	D_1	D_2	
O_1	1	2	5
O_2	3	5	10
	8	7	

(i) 32
(ii) 30
(iii) 38
(iv) 44.

(i) In 2×2 pay-off matrix $\begin{pmatrix} 6 & 1 \\ x & 5 \end{pmatrix}$ the value of the game is 5. Then the value of x is

(i) 0
(ii) 1
(iii) 3
(iv) 5.

(j) Consider the game with the following pay-off matrix :

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} p & 7 & 3 \\ -2 & p & -8 \\ -3 & 4 & p \end{bmatrix} \end{array}$$

The value of p for which the game is strictly determinable satisfies

(i) $-8 \leqslant p \leqslant -3$ (ii) $-3 \leqslant p \leqslant -2$

(iii) $-2 \leqslant p \leqslant 3$ (iv) $-8 \leqslant p \leqslant 7$.

Unit – I

2. Answer *any two* questions :

(a) A factory is engaged in manufacturing three products A, B and C which involve lethe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 2 and 1 hours respectively. Similarly they are 3, 1, 3 hours for one unit of B and 1, 3, 1 hours for one unit of C. The profits on A, B and C are ₹ 2, ₹ 3 and ₹ 4 per unit respectively. Assuming that 300 hours of lethe time, 240 hours of grinding time and 250 hours of assembling time are available, the manufacturer wants to produce the different types of items A, B, C so as to maximize his profit. Formulate the above as a L.P.P. 5

(b) Show that $x = 1, y = 3, z = 2$ is a feasible solution of the equations :

$$x + 2y - z = 5$$

$$10x + 3y + 7z = 33$$

Reduce the above feasible solution to a basic feasible solution. 1+4

(c) Solve graphically the following L.P.P. : 5

Minimize $Z = -x_1 + 2x_2$

subject to $-x_1 + 3x_2 \leqslant 10,$

$x_1 + x_2 \leqslant 6,$

$x_1 - x_2 \leqslant 2,$

$x_1, x_2 \geqslant 0.$

(d) Prove that every extreme point of the convex set of all feasible solutions of the system of equations $A\tilde{X} = \tilde{b}$, $\tilde{X} \geq \tilde{0}$, corresponds to a basic feasible solution of the system. 5

Unit – II

3. Answer *any one* question :

(a) (i) Apply simplex method to solve the following L.P.P. :

$$\text{Maximize } Z = 3x_1 + 2x_2 + x_3$$

$$\text{subject to } \begin{aligned} 5x_1 + 6x_2 + 10x_3 &\leqslant 50, \\ 5x_1 + 4x_2 &\leqslant 20, \\ 15x_1 + 4x_2 &\leqslant 30, \\ x_1, x_2, x_3 &\geqslant 0. \end{aligned}$$

(ii) Show that the L.P.P.

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{subject to } \begin{aligned} 2x_1 + x_2 &\leqslant 2, \\ 3x_1 + 4x_2 &\geqslant 12, \\ x_1, x_2 &\geqslant 0. \end{aligned}$$

has no feasible solution. 7+3

(b) (i) Solve the following L.P.P. by penalty method :

$$\text{Maximize } Z = -2x_1 + x_2 + 3x_3$$

$$\text{subject to } \begin{aligned} x_1 - 2x_2 + 3x_3 &= 2, \\ 3x_1 + 2x_2 + 4x_3 &= 1, \\ x_1, x_2, x_3 &\geqslant 0. \end{aligned}$$

(ii) Use two phase simplex method to solve the following L.P.P. :

$$\text{Minimize } Z = x_1 + x_2$$

$$\text{subject to } \begin{aligned} 2x_1 + x_2 &\geqslant 4, \\ x_1 + 7x_2 &\geqslant 7, \\ x_1, x_2 &\geqslant 0. \end{aligned}$$

5+5

Unit – III

4. Answer *any one* question :

(a) (i) Given the following L.P.P. :

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 4x_2 \\ \text{subject to} \quad & x_1 - x_2 \leqslant 1, \\ & x_1 + x_2 \geqslant 4, \\ & x_1 - 3x_2 \leqslant 3, \\ \text{and } & x_1,\ x_2 \geqslant 0. \end{aligned}$$

Using Simplex method, show that dual of the above L.P.P. has no feasible solution.

(ii) Formulate the dual of the L.P.P. given below :

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + 3x_2 + 4x_3 \\ \text{subject to} \quad & x_1 - 5x_2 + 3x_3 = 7, \\ & 2x_1 - 5x_2 \leqslant 3, \\ & 3x_2 - x_3 \geqslant 5, \\ & x_1,\ x_2 \geqslant 0 \end{aligned}$$

and x_3 is unrestricted in sign. 6+4

(b) (i) If the primal and its dual L.P.P. both have feasible solutions, then the objective function of both the L.P.P. are bounded.

[Bounded above for the maximization problem and bounded below for the minimization problem.]

(ii) Write down the dual of the L.P.P.

$$\begin{aligned} \text{Maximize } & Z = 3x_1 + 4x_2 \\ \text{subject to } & x_1 + x_2 \leqslant 10, \\ & 2x_1 + 3x_2 \leqslant 18, \\ & 0 \leqslant x_1 \leqslant 8, \\ & 0 \leqslant x_2 \leqslant 6. \end{aligned}$$

Find optimal solution of the above L.P.P. by solving its dual. 4+(2+4)

Unit – IV

5. Answer *any three* questions : 5×3

(a) Using VAM, solve the following transportation problem and find the optimal solution.

	D_1	D_2	D_3	D_4	$a_i \downarrow$
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
$b_j \rightarrow$	5	8	7	14	

(b) Find the optimal assignment from the following profit matrix :

	1	2	3	4
1	40	35	43	45
2	33	39	48	33
3	40	37	33	32
4	35	41	39	37

(c) A salesman has to visit five cities *A, B, C, D, E*. The distances (in hundred kilometers) between the cities are as follows :

	A	*B*	*C*	*D*	*E*
A	∞	3	6	2	3
B	3	∞	5	2	3
C	6	5	∞	6	4
D	2	2	6	∞	6
E	3	3	4	6	∞

If the salesman starts from city *A* and has to come back at city *A*, which route should he select so that the total distance travelled is minimum?

(d) Using Graphical method, solve the following rectangular game with the following pay-off matrix :

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	1	0	4	–1
	A_2	–1	1	–2	5

(e) Use dominance to reduce the pay-off matrix and then solve the game problem :

$$\begin{array}{cc} & \text{B} \\ \text{A} & \begin{array}{|c|c|c|c|} \hline -5 & 3 & 1 & 20 \\ \hline 5 & 5 & 4 & 6 \\ \hline -4 & -2 & 0 & -5 \\ \hline \end{array} \end{array}$$

Paper : DSE-B-1.3

(Boolean Algebra and Automata Theory)

Full Marks : 65

1. All are multiple choice questions with single correct option. Students are required to opt the correct option and justify the correct option in their own words as far as practicable. One mark for correct option and one mark for justification. There is no negative marking. 2×10

(a) A tree with N number of vertices contains

(i) $(N-1)$ edges (ii) (N^2-1) edges

(iii) N edges (iv) $(N+1)$ edges.

(b) Let $X=\{2, 3, 6, 12, 14\}$ and $\leq$ be the partial order defined by $x \leq y$ if x divides y. Number of edges in the Hasse diagram of $(X, \leq)$ is

(i) 3 (ii) 4

(iii) 5 (iv) None of these.

(c) The complement of the expression $(W'X+YZ')$ is

(i) $(W'+X)(Y'+Z)$ (ii) $(W+X')(Y'+Z)$

(iii) $(W''+X)(Y''+Z)$ (iv) $(W+X')(Y+Z')$.

(d) The following CFG

$S \to As \mid bS \mid a \mid b$, is equivalent to the regular expression :

(i) $(a+b)^*$ (ii) $(a+b)(a+b)^*$

(iii) $(a+b)^*(a+b)$ (iv) All of these.

(e) Regular expressions for all strings starts with ab and ends with bba is

(i) aba^*b^*bba (ii) $ab(ab)^*bba$

(iii) $ab(a+b)^*bba$ (iv) All of these.

(f) The minimum number of productions required to produce a language consisting of palindrome string over $\sum=\{a,b\}$ is

(i) 3 (ii) 5

(iii) 7 (iv) 6.

(g) The symmetric closure of the relation $\rho=\{(a,b)/a>b\}$ on the set of positive integers, is

(i) $\{(a, b)/a \geq b\}$ (ii) $\{(a, b)/a \leq b\}$

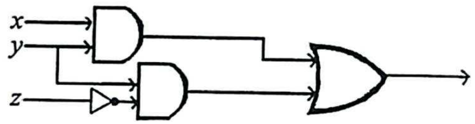
(iii) $\{(a, b)/a \neq b\}$ (iv) $\{(a, b)/a = b\}$.

(h) The output sequence Y for an AND gate with inputs $A = 110001$, $B = 101101$ and $C = 110011$ is

(i) $Y = 100001$ (ii) $Y = 001110$

(iii) $Y = 101001$ (iv) $Y = 100111$.

(i) The Boolean expression that represents the logic circuit



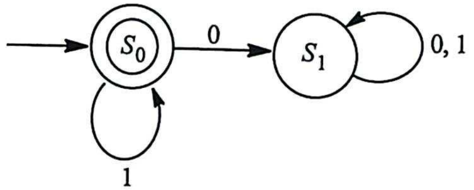
is

(i) $xy + yz$ (ii) $xy + yz'$

(iii) $(x + y)z'$ (iv) $xy + z'$.

[where z' denotes the complement of z]

(j) The language recognized by the finite-state automata M, given by



is

(i) $L(M) = \{1^n / n = 0, 1, 2, ...\}$

(ii) $L(M) = \{0^n 1^n / n = 0, 1, 2, ...\}$

(iii) $L(M) = \{0^m 1^n / m, n = 0, 1, 2, ...\}$

(iv) $L(M) = \{1, 01\}$

Unit - I

Answer *any one* question.

2. Find the reflexive transitive closure R^* of the relation $R = \{(1,2), (2,3), (3,1), (4,4)\}$. Draw a directed graph representing R^*. 2+2

3. Prove that every distributive lattice is modular. Is the converse true? Justify your answer. 2+(1+1)

Unit - II

Answer *any one* question.

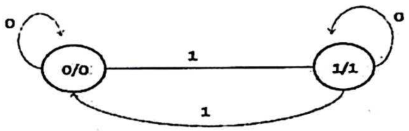
4. Simplify the logical function $y = x_1'x_2x_3' + x_1x_2x_3' + x_1'x_2x_3 + x_1x_2x_3$ by using Karnaugh map method, where x_1', x_3' are the complements of x_1 and x_3 respectively. 4

5. Construct a circuit diagram for the Boolean function— $(BC+A)\left(\overline{A}\overline{B}+\overline{C}\right)+\overline{A}\overline{B}\overline{C}$.
Using laws of Boolean Algebra, simplify the function and draw the simplified circuit. 2+2

Unit - III

Answer *any one* question.

6. A Finite State Machine (FSM) is given as in the following diagram :



(a) What is the output sequence for the given input data sequence?
0 0 1 0 1 0 1 1 0 1 1 0 1 1 0 1 1 1

(b) What is the behaviour of the above finite state machine? 2+2

7. Let $M=(Q,\Sigma,\Gamma,q_0,\delta,\gamma)$ be a Finite State Machine with transition table

	δ		γ	
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

(a) Draw the transition diagram of *M*.

(b) What is the output string, if the input string is *ababab*? 2+2

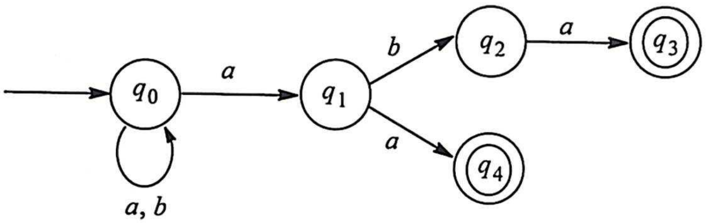
Unit - IV

Answer *any two* questions.

8. (a) Show that the language $L=\left\{0^{3m}\ 1^{2n} : m,n>0\right\}$ is not regular using Pumping Lemma.

(b) Construct a grammar which generates the language *L* described by the expression *aa*bb**. 4+3

9. Let M be the NDFA whose state diagram is givne by the following figure :



(a) Write the transition table for M.

(b) Find a directed walk with the label '$bbabbaa$' from q_0 to q_4.

(c) What are the final states?

(d) Is 'baa' in $L(M)$?

(e) Find $\delta^*(q_0, baa)$

[where $L(M)$ and δ^* have their usual meanings.]

1+1+1+2+2

10. Let $G = (V_N, \Sigma, P, S)$ be a context-free grammer, where $V_N = \{S, A\}$, $\Sigma = \{a, b\}$, $P = \{S \rightarrow Ab, A \rightarrow aAb,\ A \rightarrow \lambda\}$

(a) Write a derivation of '$aaabbbb$' from S in G.

(b) Draw a derivation tree of the derivation of G with yield $aaaabbbb$.

(c) Find the language $L(G)$ generated by this grammer.

2+2+3

Unit - V

Answer *any two* questions.

11. Find a grammer in Chomsky Normal Form equivalent to the grammer $S \rightarrow aAbB$, $A \rightarrow aA|a$, $B \rightarrow bB|b$.

6

12. Design a Turing Machine M, to recognize the language $\{1^n 2^n 3^n \mid n \geq 1\}$.

6

13. Let $L = \{a^m b^n \mid n < m\}$ be a language. Construct—

(a) a context-free grammer accepting L.

(b) a push down automata accepting L by empty store.

(c) a push down automata accepting L by final state.

2+2+2

Unit - VI

Answer ***any one*** question.

14. (a) Design the Turing machine accepting palindromes over $\{a, b\}$.

(b) Prove that union of two recursive languages is recursive. 3+4

15. (a) Solve the given PCP instance :

Top : $\{ab, b, a\}$

Bottom : $\{a, ba, b\}$

Determine if a match exists and explain your reasoning.

(b) Discuss why a general algorithm can not solve all PCP instances.

(c) Mention one field where undecidable problems like PCP pose a challenge. 4+2+1
