

2024

## MATHEMATICS — HONOURS

Paper : CC-7

(ODE and Multivariate Calculus – I)

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.* $\mathbb{R}$  denotes the set of real numbers.

Group - A

(Marks : 20)

1. Answer the following multiple-choice questions with only one correct option. Choose the correct option and justify : (1+1)×10

(a) The equation of the integral curve through the point (2, 3) corresponding to the differential equation

$$\frac{dy}{dx} = \frac{y}{x} \text{ is}$$

(i)  $3y = 2x$

(ii)  $2y = 3x$

(iii)  $y = 3x$

(iv) None of these.

(b) An integrating factor of the differential equation  $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$  is

(i)  $\frac{1}{xy}$

(ii)  $\frac{1}{x^2 + y^2}$

(iii)  $\frac{1}{x^2 y^2}$

(iv)  $\frac{1}{x^2 - y^2}$

(c) Which of the following differential equations is linear?

(i)  $5\frac{dy}{dx} + x\sqrt{y} = 5e^{4x}$

(ii)  $\frac{dy}{dx} + 5x^2 \tan y = x$

(iii)  $e^x \frac{dy}{dx} + y = e^{-x}$

(iv)  $y \frac{dy}{dx} + 5xy = \log_e x$

(d) The Wronskian of the functions  $y_1 = e^x \sin x$  and  $y_2 = e^x \cos x$  is

(i) 0

(ii)  $e^x \cos x$

(iii)  $e^x \sin x$

(iv)  $-e^{2x}$

Please Turn Over

(e) The singular solution of the equation  $p^2 + xp - y = 0$  is

- (i)  $y^2 = 4x$  (ii)  $x^2 = -4y$   
 (iii)  $x^2 + y^2 = xy$  (iv)  $xy = 4$ .

(f) Which of the following is correct for the linear differential equation

$$(3x+1)x \frac{d^2y}{dx^2} - (x+1) \frac{dy}{dx} + 3y = 0 ?$$

- (i) 0 is an irregular singular point (ii) -1 is an irregular singular point  
 (iii) -1 is a regular singular point (iv) no irregular singular point.

(g) The value of  $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin(x^2 + y^2)}{x^2 + y^2}$  is

- (i) 1 (ii) -1  
 (iii) 0 (iv) does not exist.

(h) The unit normal to the surface  $x^2 + y^2 = z$  at point (1, 2, 5) is

- (i)  $\frac{2}{\sqrt{21}} \bar{i} - \frac{4}{\sqrt{21}} \bar{j} - \frac{\bar{k}}{\sqrt{21}}$  (ii)  $\frac{-2}{\sqrt{21}} \bar{i} + \frac{4}{\sqrt{21}} \bar{j} - \frac{\bar{k}}{\sqrt{21}}$   
 (iii)  $\frac{2}{\sqrt{21}} \bar{i} + \frac{4}{\sqrt{21}} \bar{j} - \frac{\bar{k}}{\sqrt{21}}$  (iv)  $\frac{2}{\sqrt{21}} \bar{i} + \frac{4}{\sqrt{21}} \bar{j} - \frac{\bar{k}}{\sqrt{21}}$ .

(i) If  $z = \frac{x+y}{x-y}$ ,  $x \neq y$ , the value of  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$  is

- (i)  $2 \frac{x+y}{x-y}$  (ii)  $2 \frac{x+y}{(x-y)^2}$   
 (iii)  $\frac{2}{x-y}$  (iv)  $\frac{-2}{x-y}$ .

(j) For the function  $f(x, y) = x^2 - y^3 - x^2y + y$ , the point  $\left(0, \frac{1}{\sqrt{3}}\right)$  is

- (i) not a critical point (ii) a saddle point  
 (iii) a point of local minimum (iv) a point of local maximum.



(3)

B(3rd Sm.)-Mathematics-H/CC-7/CBCS

## Group - B

(Marks : 30)

Answer *any six* questions.

2. (a) State the existence and uniqueness theorem for the initial value problem  $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ .

(b) Solve the equation  $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$ . 2+3

3. Find the value of the constant  $\lambda$  for which the differential equation  $(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$  is exact. Hence solve the equation. 2+3

4. Solve :  $x^2y - x^3 \frac{dy}{dx} = y^4 \cos x$ . 5

5. Reduce the equation  $y^2(y - xp) = x^4p^2$  to Clairaut's form by the substitution  $x = \frac{1}{u}, y = \frac{1}{v}$  and hence solve it. Find the singular solution, if it exists. 2+2+1

6. Find the solution of the differential equation  $(D^2 - 2D + 1)y = xe^x$   $\left(D \equiv \frac{d}{dx}\right)$  by the method of variation of parameters. 5

7. Solve the differential equation  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 8x^2 + 3 + 2\cos 2x$  by the method of undetermined coefficients. 5

8. Find the general solution of the following Euler-Cauchy differential equation :

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$$
5

9. Find  $x$  and  $y$  from the system of equations :

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

5

Please Turn Over

(1385)

10. Determine the nature and stability of the critical point  $(0, 0)$  of the following system :

$$\frac{dx}{dt} = 3x + 2y$$

$$\frac{dy}{dt} = x + 2y$$

Also draw rough sketch of the corresponding phase portraits.

3+2

11. Solve the equation  $\frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 3y = 0$  about the ordinary point  $x = 0$ .

5

### Group - C

(Marks : 15)

Answer *any three* questions.

12. (a) Show that the set  $S = \{(x, y) \in \mathbb{R}^2 : 0 \leq x < 1, 0 \leq y < 1\}$  is neither open nor closed.  
 (b) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} y + x \sin\left(\frac{1}{y}\right), & \text{if } y \neq 0 \\ 0 & \text{if } y = 0 \end{cases}$$

Show that  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$  but  $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$  does not exist.

2+3

13. Examine the existence of maxima or minima of the function  $f(x, y) = x^2 + y^2 + (x + y + 1)^2$ .

5

14. If  $u = f(x^2 + 2yz, y^2 + 2zx)$ , prove that

$$(y^2 - zx) \frac{\partial u}{\partial x} + (x^2 - yz) \frac{\partial u}{\partial y} + (z^2 - xy) \frac{\partial u}{\partial z} = 0.$$

5

15. Find the maximum or minimum of the function  $f(x, y) = xy$ , subject to the condition  $5x + y = 13$ , using the method of Lagrange's Multipliers.

5

16. Find the directional derivative of the function  $\phi(x, y, z) = x^2yz + 4xz^2$  at the point  $(1, -2, 1)$  in the direction of the vector  $2\hat{i} - \hat{j} - 2\hat{k}$ . Also find the greatest rate of increase of  $\phi$ .

3+2